BOOK V

OF THE HARMONY OF THE WORLD

by

JOHANNES KEPLER

ON THE MOST PERFECT HARMONY OF THE HEAVENLY MOTIONS, and on the origin from the same of the Eccentricities, Semidiameters, and Periodic Times. According to the precepts of the most thoroughly corrected astronomical teaching of the present day, and the hypotheses of Copernicus, but also those of Tycho Brahe, one or the other of which are today publicly accepted as true, superseding those of Ptolemy.

Galen, *On The Function of Parts*, Book III,[1]

"I am undertaking a holy discourse, a true hymn to God our Creator, believing that piety consists not in my sacrificing a great many hecatombs of bulls to Him, nor in my offering innumerable scents and spices; but in my first learning myself, and then teaching others, both His greatness in wisdom, His greatness in power, and His nature in goodness. For I take as a demonstration of the most absolute goodness the wish to embellish all things with the greatest possible adornment, and to envy no man his goods; and I honor Him as good accordingly; and furthermore I take it as a demonstration of outstanding wisdom, to seek out everything by which He might be adorned to the utmost; and finally as a demonstration of irresistible power, to carry out all that He had ordained."

With Imperial privilege for fifteen years.

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[1] The quotation is from Galen, *De usu partium corporis humani*. See the edition of Georg Helmreich (Leipzig, 1907), vol. 1, p. 174. Kepler gives the Greek with his own Latin translation.

Introduction

The discovery which I foretold twenty two years ago,[2] when I first discovered the five solid figures between the celestial spheres; the discovery which I firmly set my mind to, before I had seen Ptolemy's *Harmony*; the discovery which I promised to my friends in giving this fifth book its title, before I was sure of the fact itself[3]; the discovery which sixteen years ago in a published work I insisted should be sought for[4]; the discovery for the sake of which I applied the best part of my life to astronomical studies, I visited Tycho Brahe, and I chose Prague as my seat; that discovery at last, with God the Best and Greatest, who had inspired my thought and had aroused this mighty ambition, also prolonging my life and the vigor of my talents, and supplying the rest of my requirements through the generosity of two Emperors, and of the chief men of this province of Upper Austria, on the completion of my previous work in the province of astronomy to a sufficient extent, at last, I say, I have brought that discovery into the light, and have most truly grasped beyond what I could ever have hoped: that the whole nature of harmony, to its full extent, with all its parts, as expounded in Book III, is to be discovered among the celestial motions. It is to be discovered indeed not in the way which I had mentally conceived (and this is not the least part of my joy), but in a totally different way,[5]

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[2] In the *Mysterium cosmographicum*, Chapter 18: Duncan (1981), 181. Writing in his notes to the second edition, published after the *Harmonice mundi*, Kepler remarks of his forecast:

> The amounts of the eccentricities have been investigated by me from the observations of Brahe; the causes of the eccentricities have been made clear in the *Harmonice*; and you can see that arcs which agree with the motions in all respects have been inferred, not indeed from the five figures alone, but chiefly from the causes of the eccentricities (the harmonies) (ibid., 187).

[3] Kepler described the principal ideas of the *Harmonice mundi* in letters to Edmund Bruce, Maestlin, and Herwart von Hohenburg written in the autumn and winter of 1599. In one of the letters to Herwart (KGW 14, p. 100), he declared his plan to write a cosmographic dissertation evidently based on the quadrivium, which would have the title *De harmonice mundi* and consist of five parts, the fifth of which would explain the origins or causes of the periodic motions of the planets.

[4] This is a reference to *De fundamentis astrologiae certioribus* (1602), thesis 37. See the translation by J.V. Field (1984a), 250. Cf. the translation by M.A. Rossi (1979), 97.

[5] In the *Mysterium cosmographicum*, Kepler had supposed the planetary distances to have been determined solely by the interpolation of the five Platonic solids between the planetary spheres and he had sought, in Chapter 12, to fit the musical harmonies to the solids. See Duncan (1981), 133. Now he supposed that the interpolation of the solids gave only a rough guide (a preliminary sketch of the cosmos, so to speak), while the distances were really determined in accordance with the harmonic relations of the motions of the planets. Of his earlier attempt to relate the distances to the mo-

and also at the same time a quite outstanding and perfect way. In addition during this intermediate period, in which the extremely laborious restoration of the motions held me in suspense, my appetite was particularly intensified and my purpose stimulated by the reading of the *Harmony* of Ptolemy, which was sent over to me in manuscript by that great man, born for the advancement of philosophy and indeed of every kind of scholarship, Johannes Georg Herwart, Chancellor of Bavaria.[6] There I found unexpectedly, and to my great wonder, that

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tions, set out in the *Mysterium cosmographicum*, Chapter 20, Kepler comments in the second edition:

> This is the proper subject . . . of Book V of the *Harmonice* . . . Although I had not yet attained what I was seeking, yet a number of principles were introduced which then seemed to me to be in agreement with the nature of things, and quite certain, and which I have found very useful throughout the last 25 years, especially in the *Commentaries on the motion of Mars*, Part IV (see Duncan (1981), 203).

[6] Kepler received the manuscript of Ptolemy's *Harmonica* together with that of Porphyry's commentary in 1607 (KGW 15, p. 415). Ptolemy treats the harmony of the planets in Book III, Chapters 10 to 16, though in the case of Chapters 14 to 16 only the headings of the chapters survive in the Greek text. Commenting on Chapters 10 to 13, Kepler declares that Ptolemy runs riot in using poetic or rhetorical comparisons, since what he compares are not real objects in the heavens. In Chapter 10, Ptolemy compares the longitudinal motion of the planets, in accordance with which the path from rising to setting is traversed, with a continuous melody rising and falling in pitch, where the highest note corresponds to culmination and the lowest to the rising and setting points. In Chapter 11, the variations in motions of the planets arising from their changes in distance are correlated with the three harmonic genera, the enharmonic with the least velocity, the diatonic with the greatest velocity and the chromatic with the mean velocity. Reasons are given. Thus, in the chromatic genus, the note lichanos approximately halves the tetrachord, as the mean velocity lies halfway between the extremes. The enharmonic corresponds to the least velocity, because in this genus, the two smaller intervals are together less than the third, while the diatonic corresponds to the greatest velocity, because in this genus, any two intervals are together greater than the third. Ptolemy also appends other reasons, associating the moral characteristics of the genera with the emotional influences of more or less rapid motions. In Chapter 12, the motions of the planets in latitude are correlated with the modulations between tones or modes. The reason he gives is that the modulations do not involve a change of genus, while the motion of a planet in latitude does not involve a perceptible change in velocity. The Dorian mode, which occupies a central position among the modes, is correlated with a position of the planet on the celestial equator. The extremes of the modes—Mixolydian and Hypodorian—correspond to the most northerly and southerly positions, while the other four correspond to parallels of latitude between the celestial equator and the tropics. In Chapter 13 Ptolemy considers an analogy between the tetrachords and the aspects of planets with the sun, a comparison that Kepler describes as consistent but absurd (KOF, vol. 5, p. 386).

Kepler himself reconstructed Ptolemy's Chapters 14 to 16 in accordance with Ptolemy's principles in order to supply a suggestion of coherence of the missing part of Ptolemy's work with the rest. In his notes, besides explaining his own objections, he supplies in a sense the missing part of Porphyry's commentary.

In Chapter 14, Ptolemy compares the notes of the musical scale with the primary spheres of his planetary system. Kepler comments that the comparison should have been made with the magnitudes of the spheres but these, as he had shown, bear no relation to the consonances. In Chapter 15, Ptolemy asks how the proportions of the almost the whole of his third book was given up to the same study of the celestial harmony, one thousand five hundred years before. Yet at that period much was still lacking in astronomy, and Ptolemy, by an unfortunate attempt, might have brought others to despair, as like Cicero's Scipio[7] he seemed to have related some pleasant Pythagorean dream rather than to have assisted philosophy; but I was emphatically strengthened in the pursuit of my purpose, on the one hand by the crudity of the ancient astronomy, and on the other by the actual agreement, precise in every detail, of both views, after an interval of fifteen centuries. For why elaborate? The very nature of things was setting out to reveal itself to men, through interpreters separated by a distance of centuries.[8] This identity of conception, on the conformation of the world, in the minds of two men who had given themselves wholly to the study of nature, was the finger of God, to borrow the Hebrew phrase, since neither had guided the other to tread this path. Now, eighteen months after the first light, three months after the true day, but a very few days after the pure Sun of that most wonderful study began to shine, nothing restrains me[9]; it is my pleasure to yield to the inspired frenzy, it is my pleasure to taunt mortal men with the candid acknowledgment that I am stealing the golden vessels of the Egyptians[10] to build a tabernacle to my God from them, far, far away from the boundaries of Egypt. If you forgive me, I shall rejoice; if you are enraged with me, I shall bear it. See, I cast the die, and I write the book. Whether it is to be read by the people of the present or of the future makes no difference: let it await its reader for a hundred years, if God Himself has stood ready for six thousand years for one to study him.

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motions of the planets can be derived through numbers. Here, Kepler remarks, Ptolemy could not succeed, because in his system of astronomy, he only had the apparent motions (as viewed from the earth): indeed nothing more than optical illusions. Finally, in Chapter 16, Ptolemy attempts to derive the basic principles of astrology concerning benign and malignant planets through the celestial harmonies. Kepler rejects the basic idea of this chapter as a misguided enterprise. For astrology is concerned with the effects of the planets on the earth, whereas Kepler's celestial harmonies are related to the sun, not the earth.

[7] Kepler made use of Macrobius' commentary on Cicero's *Somnium Scipionis*, in his reconstruction of Ptolemy's *Harmonica*, Book III, Chapter 16 (KOF, vol. 5, pp. 410–412).

[8] The two interpreters are Ptolemy and Kepler himself. Notwithstanding his fundamental disagreements with Ptolemy, Kepler admired his predecessor's commitment to the idea of a celestial harmony. Also Kepler emphasizes that his own ideas were conceived independently of Ptolemy.

[9] Thus Kepler dates his first tentative steps to the discovery of the third (harmonic) law to the end of 1616, when he completed the *Ephemerides* for 1617. On 8 March 1618, he had the law within his grasp but rejected it as a result of an error in calculation. Less than three months later, and a few days before writing his account, he discovered the law on 15 May 1618.

[10] Kepler here alludes to the story of the Israelites who stole articles of silver and gold from the Egyptians (Exodus 12, vv. 35–36) and after their flight from Egypt used them to build a tabernacle (Exodus 25, vv. 1–8).

THE CHAPTERS OF BOOK V.

The Appendix contains:[11]

I. A version of the third book of Ptolemy's Harmony, from Book III, which deals with the same material.

II. A completion of the Ptolemaic text, of Ptolemy's last three chapters, in which Ptolemy supplied only the lemmas.

III. Notes on this part of the Harmony, in which I explain the author, refute him, and compare his discoveries or attempts with my own.

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[11] Kepler dispensed with this plan and confined his appendix to a summary.

As I am on the point of dealing with these matters, it pleases me to impress on my readers the sacred exhortation of Timaeus, the Gentile philosopher, when he was starting on the same matters, which should be acknowledged by Christian men with the greatest wonder, and shame if they do not imitate him. It is as follows: (Greek text)

> "But, Socrates, everyone certainly who has even a modicum of sense, on embarking on any undertaking whether small or great, always calls on God; and for us who are about to hold a discussion on the Whole, if we are not completely astray, it is essential to invoke all the gods and goddesses in prayer, speaking according to their will chiefly, but secondarily to yours."

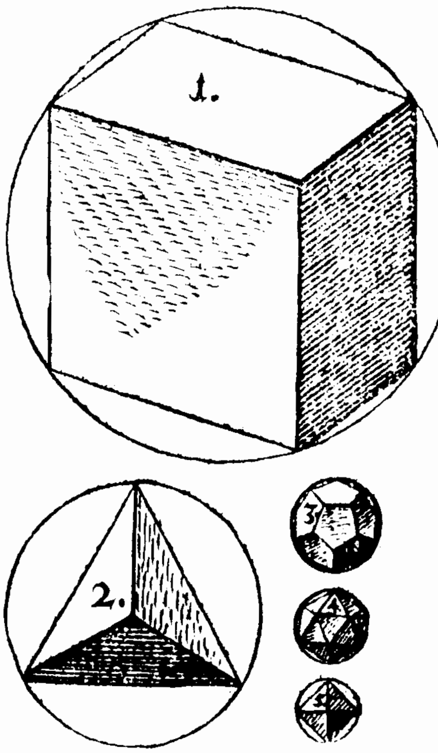
[12]For indeed, Socrates, since everyone who has even the smallest amount of sense, whenever they embark on any undertaking whether easy or difficult, always calls on God, it is essential for us, whose intention is to debate the totality of things, unless we stray completely from all sane judgement, to pray to the gods and goddesses, our desires in our hearts, with one accord, so that we may utter what will be pleasing and acceptable, first to them chiefly, then secondly to you also.

[12] Plato, *Timaeus*, 27C. Kepler's Latin translation.

Chapter I.

On the Five Regular Solid Figures.

We have stated in Book II how the regular plane figures fit together to form the solid figures.[13] That is, we spoke there of the five solid figures (among others) in connection with the plane figures. Now it was shown there that the number of the solid figures is fivefold; and in addition it was shown why they were named Cosmic by the Platonists, and to which element, and on account of what property, each of them was related. Now at the entry to this Book we shall have to deal again with these figures, for their own sake and not because of the plane figures, that is as much as suffices for the harmony of the heavens. The reader will find the rest in Part II[14] of the *Epitome of Astronomy*, Book IV.



The circles circumscribing the vertices sketch out the spheres; but they are to be conceived as slightly larger than these circles, except in the case of 5, so that in fact they just graze all the angles of the figure. The ratio of size has been kept in this diagram, so that the sphere of 5 can be inscribed in 4, and the latter's sphere again in figure 3, and 3's in 2, and finally the sphere of 2 in the cube, 1, touching all its six faces at their centers.

Therefore, from the *Mysterium Cosmographicum* (*The Secret of the Universe*)[15] I here give a brief account of the order of the five figures in the universe, three of them being primary and two secondary. For the cube, 1, is the

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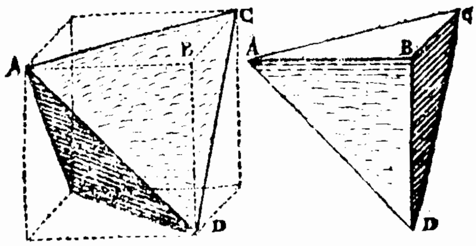
[13] On the meaning of congruence in this context, see Book II, definitions 1–6.

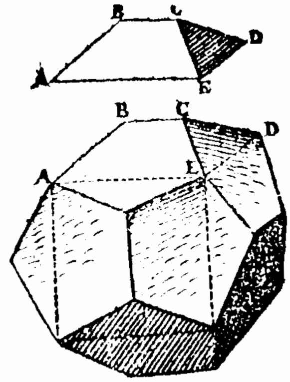
[14] This should be Part I.

[15] When he was formulating the polyhedral hypothesis relating the planetary distances, Kepler first ordered the solids by comparing the differences between the radii of their circumscribed and inscribed spheres with the intervals between the planets. Then in Chapters 3 to 8, he gave *a priori* reasons for the order indicated by the data. The major division of the solids is into two classes, primary and secondary. The primary, consisting of the cube, tetrahedron, and dodecahedron, have faces of different shapes and vertices common to three faces, the minimum number needed to form a solid angle. The secondary, consisting of the octahedron and icosahedron, have faces of the same shape and vertices common to four or five faces. As the abode of Man, the Earth occupies a privileged position between the two classes, which may be described as a kind of geocentrism of importance. Further points of distinction between the solids are given in the *Epitome astronomiae Copernicanae*, Book IV, Part 1, Chapter 3.

outermost and the largest, because it is first in order of generation, and relates to the Whole, by the very form of its generation. There follows the tetrahedron, 2, as a part established by division of the cube,

In the first figure the tetrahedron ACDF appears hidden in the cube, in such a way that any face of the tetrahedron, such as ACD, is covered by one vertex of the cube ACDB. In the second figure the cube AED appears hidden inside the dodecahedron, in such a way that any face of the cube, such as AED, is covered by two vertices of the dodecahedron or by the pentahedron ABCDE which is divisible into three dissimilar tetrahedra by the two planes DCA and ABD.

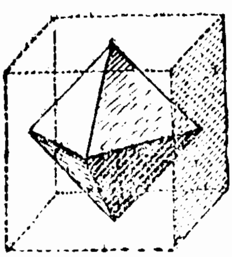


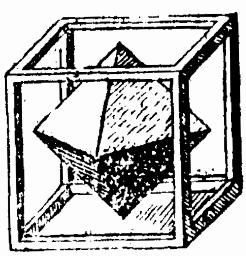


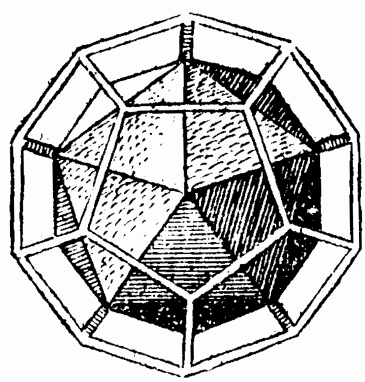
but also itself primary, with a trilinear solid angle, like the cube. Inside the tetrahedron is the dodecahedron, 3, the last of the primaries, which of course is like a composite figure made up of parts of the cube, like a tetrahedron, that is, from irregular tetrahedra, by which the cube inside it is covered. Next after it is the icosahedron, 4, on account of its similarity, the last of the secondaries which adopt a solid angle made up of more than three lines. The innermost is the octahedron, 5, which is like the cube, and is the first figure among the secondaries, to which the first place among the inner figures is due, inasmuch as it can be inscribed, just as the first place among the outer figures belongs to the cube because it can be circumscribed.

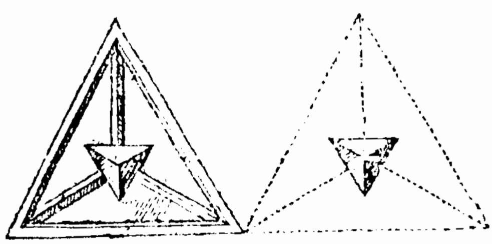
However, there are two notable marriages, so to speak, of these

Here you see the octahedron inscribed in the cube, the icosahedron in the dodecahedron, the tetrahedron in the tetrahedron.









figures, by combination from the two classes: the males, the cube and the dodecahedron from the primaries, the females, the octahedron and the icosahedron from the secondaries.[16]

[16] These sexual properties of the polyhedra are introduced here by Kepler for the first time. Evidently, the vertices are the masculine tokens of sex and the faces

In addition to these there is one which is, so to speak, celibate or hermaphrodite, the tetrahedron, because it is inscribed in itself, just as the feminine ones are inscribed in, and so to speak subject to, the males, and have the female tokens of their sex opposite to the masculine ones, or in other words the angles to the plane faces.

Furthermore, as the tetrahedron is an element, the entrails and, so to speak the rib of the male cube, so the octahedron, the female, is an element and a part of the tetrahedron, by another reckoning[17]: thus the tetrahedron is the go-between in this marriage.

The chief difference between the couples or families consists in the following: that the relation between the cubic family is indeed expressible, for the tetrahedron is one third of the cubic solid, the octahedron half of the tetrahedric solid, and a sixth of the cube.[18] However, the proportion of the dodecahedric marriage is inexpressible indeed, but divine.[19]

The conjunction of these two words warns the reader to be careful about their significance. For the word "inexpressible" here does not of itself denote any nobility, as elsewhere in theology and divine matters; but it denotes an inferior condition. For there are in geometry, as has been stated in Book I, many inexpressibles which do not thereby also participate in divine proportion. Now what divine proportion is (which should rather be called sequential) must be looked for in Book I. For other proportions require their own four terms, and continuous proportion three: the divine proportion also needs a particular property of its terms, in addition to that of the proportion itself, that is to say that the two lesser terms, as parts, make up the greater term, as whole.

Therefore, to the same extent as this dodecahedric marriage has lost by using an inexpressible proportion, it also gains on the other hand because the inexpressible serves the divine.

This marriage also has a solid star,[20] the generation of which is

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the feminine ones. It may be noted that the male polyhedra have more vertices than faces. The female polyhedra, on the other hand, have more faces than vertices, while the hermaphrodite has the same number of each. In the case of each marriage, the male member has the same number of vertices as the female member has faces, so that, when the female figure is inscribed in the male, the male and female tokens face each other. Also, it may be noted that, in the case of each related pair of polyhedra, the ratio of the radii of the circumscribed and inscribed spheres is the same for both.

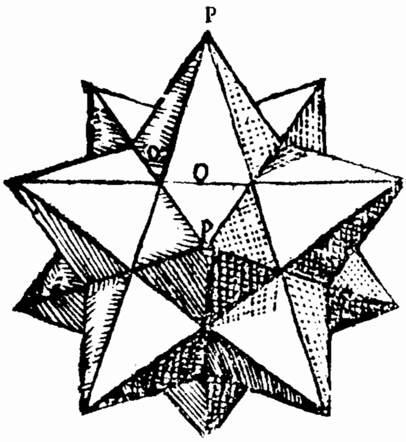
[17] The mid-points of the edges of the tetrahedron form the vertices of an octahedron.

[18] That is, the volume of the tetrahedron is one third of the volume of the cube, and so on.

[19] That is, the division in mean and extreme ratio, or golden section. See Book I, definition 26.

[20] The star-polyhedra were regarded by Kepler simply as bodies derived from the Platonic dodecahedron and icosahedron rather than additions to the class of fundamental regular polyhedra. For this reason, he did not modify his geometrical

from the continuation of five faces of the dodecahedron so that they all meet at a single point. See their generation in Book II.[21]



Lastly, we must note the proportion of the spheres circumscribing them to those inscribed in them, as in the tetrahedron it is expressible, as 100000 to 33333 or as 3 to 1; in the cubic marriage it is inexpressible, but the radius of the inscribed circle is expressible in square, as the root of one third of the square of the diameter, that is to say as 100000 to 57735; in the dodecahedric marriage it is clearly inexpressible, as 100000 to 79465; in the star it is as 100000 to 52573,[22] half the side of the icosahedron, or half the distance between the two semidiameters.

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explanation of the planetary distances in terms of the Platonic solids in order to take account of the two new polyhedra. See J.V. Field (1979a), 111.

[21] See Book II, proposition 26.

[22] In the case of the star, the ratio of the radii of the circumscribed and inscribed spheres is $\sqrt{5}:1$, so that Kepler's result should have been 100,000 to 44,721. The ratio given by Kepler is in fact the ratio of the radius of the circumscribed sphere to that of the sphere through the mid-points of the edges of the dodecahedron forming the nucleus. Kepler correctly calculates the radius of the latter sphere to be half the edge of the icosahedron formed by the points of the star (not the icosahedron in the dodecahedron mentioned earlier); in other words, half the distance between adjacent points of the star, or as he puts it, half the distance of the radii (that is, of their extremities). The exact value of the ratio intended by Kepler is $\frac{1}{2}\sqrt{2(5 + \sqrt{5})}:1$.

Chapter II.

On the Relationship of the Harmonic Proportions to the Five Regular Figures.[23]

This relationship is diverse and manifold. However, there are in the main four degrees. For either the token of the relationship is taken solely from the external appearance which the figures have; or along with the actual construction of the edge the same proportions emerge which are also harmonic; or they result from figures which have already been constructed, either separately or in conjunction; or finally they are equal to or close to the proportions of the spheres of the figure.

In the first degree the proportions of which the characteristic or major term is 3 have an affinity with the triangular face of the tetrahedron, octahedron, and icosahedron; those of which the major term is 4, on the other hand, with the square face of the cube; and those of which it is 5 with the five-sided face of the dodecahedron.

This similarity of the face can also be extended to the minor term of the proportion. Thus wherever the three-fold is found next to a term in the proportion of continuous doubling,[24] that proportion is

[23] Although the harmonic ratios have their origins in the properties of the regular plane figures, it was important for Kepler to establish a relationship of the harmonies with the regular polyhedra. As formulated in the *Mysterium cosmographicum*, the polyhedral hypothesis clearly needed adjustment, but Kepler still regarded the regular polyhedra as the basic archetype (a kind of preliminary outline) for the spacing of the planets in the cosmos. Moreover, as he remarked in the preface of the *Epitome astronomiae Copernicanae*, the archetype gave an *a priori* reason for the number of the planets and this made it clear that the world had been created. Although there were small discrepancies between the planetary distances predicted by the polyhedral hypothesis and those derived from observations, these did not affect the hypothesis as a whole. In particular, they did not indicate the need for any change in the order of interpolation between the planetary spheres that Kepler had postulated on the basis of observations and established by *a priori* reasons. For, as he explains in the *Epitome* (Book IV, Part 1, Chapter 3), no interval between two neighboring planets approaches nearer to the ratios of the spheres of another figure than the intervals which have been ascribed with the best of reasons to the two planets.

In answer to the question why there should be any discrepancies, Kepler explained that the archetype of the moveable world was constituted not only of the five solids but also of the harmonic proportions. Since this musical ornamentation needed a difference of movement in any planet (a difference made by the variation of the interval between the planet and the sun) and since the quantity of variation was required to be different for different planets, it was necessary that some small amount should be taken away from the intervals which are exhibited by the figures as constant, and that it should be left to the freedom of the composer to represent the harmonies of movement.

[24] That is, some power of 2.

considered to have affinity with the three figures first named, such as 1:3 and 2:3 and 4:3 and 8:3, and so on. Where on the other hand there is the fivefold, that proportion is to be completely appropriated to the dodecahedric marriage, such as 2:5 and 4:5 and 8:5, and similarly 3:5 and 3:10 and 6:5 and 12:5 and 24:5. The relationship will be less plausible if the sum of the terms should express this similarity, as in the case of 2:3 the terms added together make 5, as if therefore, 2:3 would have affinity with the dodecahedron.

Relationship on account of the external appearance of the solid angle is similar. It is trilinear in the primaries, quadrilinear in the octahedron, quinquelinear in the icosahedron. Thus if one term of the proportion participates in the threefold, the proportion will have affinity with the primary solids; but if it participates in the fourfold, with the octahedron; and lastly if in the fivefold, then with the icosahedron. But in the case of the feminine ones this relationship appears more attractive because the appearance of the angle is also adopted by the characteristic figure, hidden inside, the square in the octahedron, the pentagon in the icosahedron.[25] Thus 3:5 on account of both would belong to the icosahedric sect.

The second degree of relationship, which is based on origin, should be conceived as follows. First, there are some harmonic proportions of numbers which have affinity with one marriage or family, that is to say the individual perfect proportions with the cubic family. On the other hand there is a proportion which is never expressed by whole numbers, and is only demonstrated in numbers by a long series of them which gradually approach it. This proportion is called "divine," insofar as it is perfect; and it reigns in different ways through the dodecahedric marriage. Hence the following harmonies begin to sketch out this proportion: 1:2 and 2:3 and 3:5 and 5:8. For it is most imperfectly in 1:2, and most perfectly in 5:8, and would be more perfect if onto 5 and 8 added together, making 13, we were to superimpose 8, if that were not already ceasing to be harmonic.[26]

Moreover, to establish the edge of the figure the diameter of the globe[27] must be divided. The octahedron demands its division into two, the cube and the tetrahedron into three, and the dodecahedric marriage its division into five. Therefore, the proportions are distributed among the figures in accordance with these numbers, which express the proportions. Also the square of the diameter is divided, or

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[25] This means that the quadrilinear solid angle of the octahedron and the quinquelinear solid angle of the icosahedron have the same form respectively as the square in the octahedron and the pentagon in the icosahedron.

[26] The ratios quoted by Kepler are those of successive terms of the Fibonacci series. If continued indefinitely, the ratios would approach that of the golden section associated with the Pythagorean star-pentagon; that is, the divine proportion of the dodecahedric marriage.

[27] That is, the circumscribed sphere.

the square of the edge of the figure is formed, from a certain portion of it. And then the squares of the edges are compared with the square of the diameter, and establish proportions as follows: for those of the cube, 1:3, of the tetrahedron 2:3, of the octahedron 1:2. Hence for the combined pairs, for those of the cube and tetrahedron 1:2, for those of the cube and octahedron 2:3, for those of the octahedron and tetrahedron 3:4. The sides of the dodecahedric marriage are inexpressible.

Third, the figures already established give rise to harmonic proportions in various ways. For either the number of sides of the face is compared with the number of edges of the whole figure, and the following proportions emerge: for the cube 4:12 or 1:3, for the tetrahedron 3:6 or 1:2, for the octahedron 3:12 or 1:4, for the dodecahedron 5:30 or 1:6, for the icosahedron 3:30 or 1:10. Or the number of sides of the face is compared with the number of faces: then the cube gives 4:6 or 2:3, the tetrahedron 3:4, the octahedron 3:8, the dodecahedron 5:12, the icosahedron 3:20. Or the number of edges or angles of the face is compared with the number of solid angles: and the cube gives 4:8 or 1:2, the tetrahedron 3:4, the octahedron 3:6 or 1:2, the dodecahedron with its wife 5:20 and 3:12, that is 1:4. Or the number of faces is compared with the number of solid angles; and the cubic marriage gives 6:8 or 3:4, the tetrahedron the proportion of equality, the dodecahedric marriage 12:20 or 3:5. Or the number of all the sides is compared with the number of solid angles; and the cube gives 8:12 or 2:3, the tetrahedron 4:6 or 2:3, the octahedron 6:12 or 1:2, the dodecahedron 20:30 or 2:3, the icosahedron 12:30 or 2:5.

The solids are also compared with each other. If the tetrahedron is concealed in the cube, and the octahedron in the tetrahedron and cube, by inscribing them geometrically, the tetrahedron is a third of the cube, the octahedron is half the tetrahedron, and a sixth of the cube, so that the octahedron which is inscribed in the globe is also a sixth part of the cube which circumscribes the globe. The volumes of the remaining solids are inexpressible.

The fourth kind or degree of relationship is more appropriate to the present work,[28] as what is sought is the proportion of the

[28] The importance of this degree of relationship lies in the fact that it enables Kepler to explain the discrepancies in the polyhedral hypothesis. For while the Creator had been free to adjust the planetary distances in accordance with the requirements of the musical ornamentation represented by the harmonic movements, these adjustments were not the results of an arbitrary choice but had a rational basis. Kepler describes these discrepancies at length in Book V, Chapter 3, and Chapter 9, proposition 47, but more succinctly in the *Epitome*, Book IV Part 1, Chapter 3. In the case of the tetrahedron, the ratio of the radii of the circumscribed and inscribed spheres is expressible. Correspondingly, the tetrahedric planets (Jupiter and Mars) imitate this figure almost exactly; that is, in approximately the extremities of the intervals. In the case of the cube and octahedron, the ratios of the radii of the circumscribed and inscribed spheres are only expressible in square. Correspondingly, the cubic planets

spheres inscribed in the figures to the spheres circumscribing them, and what is calculated is the harmonic proportions which come close to them. For only in the tetrahedron is the diameter of the inscribed sphere expressible, that is as one third of the circumscribed sphere; but in the cubic marriage the proportion, which is unique to that case, is similar to lines which are expressible only in square. For the diameter of the inscribed sphere is to the diameter of the circumscribing sphere in the semitriple proportion.[29] And if you compare the actual proportions with each other, the proportion of the tetrahedric spheres is the square of the proportion of the cubic spheres. In the dodecahedric marriage, the proportion of the spheres is again unique, but inexpressible, a little greater than 4:5. Therefore, the harmonic proportions which are close to the proportion of the cubic and octahedric spheres are the following: 1:2 as the next greatest, and 3:5 as the next smallest; while the harmonies which are close to the proportion of the dodecahedric spheres are 4:5 and 5:6, the next smallest, and 3:4 and 5:8, the next greatest.

But if for particular reasons 1:2 and 1:3 are to be appropriated to the cube, as the proportion of the cube's spheres to the proportion of the tetrahedron's spheres, so the harmonies 1:2 and 1:3, which are allotted to the cube, will be to 1:4 and 1:9 which have to be allotted to the tetrahedron, if indeed it is right to use this analogy; for these proportions are the squares of the harmonies mentioned. And because 1:9 is not harmonic, its place will be taken by the nearest harmonic, 1:8, for the tetrahedron. But to the dodecahedric marriage, using this analogy, will belong approximately 4:5 and 3:4. For just as the proportion of the cubic spheres is approximately the cube of that of the dodecahedric, so also the cubic harmonies 1:2 and 1:3 are approximately the cubes of the harmonies 4:5 and 3:4. For 4:5 cubed is 64:125; and 1:2 is 64:128. Similarly 3:4 cubed is 27:64, and 1:3 is 27:81.

(Saturn and Jupiter) and the octahedric planets (Venus and Mercury) are less exactly like their figures, because the extreme intervals recede from the ratio of the figures, though the mean intervals are in agreement. Finally, the ratios of the radii of the circumscribed and inscribed spheres of the dodecahedron and the icosahedron are inexpressible. Correspondingly, the intervals of the dodecahedric planets (Mars and Earth) and the icosahedric planets (Earth and Venus) abandon the ratios of their figures, though they do not approach closer to those of any other figure.

[29] That is, the ratio $1:\sqrt{3}$.

Chapter III.

Summary of Astronomical Theory, Necessary for the Study of the Heavenly Harmonies.

To start with, readers should know that the ancient astronomical hypotheses of Ptolemy, in the way in which they have been expounded in the *Theoricae* of Peurbach[30] and in the other writers of Epitomes,[31] have been totally excluded from this discussion, and put out of mind; for they do not convey truthfully either the arrangement of the bodies in the world or the commonwealth of the motions.

In their place I cannot do other than substitute solely the opinion of Copernicus on the world, and, if it were possible, persuade everyone to believe it. Yet it is still a new idea to the common herd of scholars, and a doctrine which to many is quite absurd to hear, that the Earth is one of the planets, and is carried among the stars round an unmoving Sun. Therefore, those who are shocked at the novelty of this opinion should know that these speculations about harmonies also find a place in the hypotheses of Tycho Brahe, because that author has everything else which relates to the arrangements of the bodies and the combination of their motions in common with Copernicus.[32] It is only the Copernican annual motion of the Earth which he transfers to the whole system of the planetary spheres, and to the Sun, which occupies the middle by the agreement of both authors. For by this transference of the motion it nevertheless comes about that the Earth,

The hypotheses of Copernicus.

Hypotheses of Tycho Brahe.

Comparison of the two.

[30] G. Peurbach, *Theoricae novae planetarum*. See the English translation by E.J. Aiton (1987).

[31] The best known is *Epytoma Joannis de Monte Regio in Almagestum Ptolomei*; reprinted in F. Schmeidler (1972), 55–274.

[32] Both here and on the title page of Book V Kepler emphasizes that the theory he describes is equally applicable to the systems of Copernicus and Tycho Brahe, which he regarded as kinematically equivalent. In 1616, Copernicus' *De revolutionibus* had been suspended pending corrections by the Holy Congregation of the Index and on 10 May 1619, the first part of Kepler's *Epitome astronomiae Copernicanae*, published in 1618, was also placed on the Index. Johannes Remus Quietanus, the Imperial physician-in-ordinary, who communicated this news to him, remarking that Galileo would like to have a copy (KGW 17, p. 362), and also the Venetian Vincenzo Bianchi (KGW 17, p. 319), assured him that his books could still be read in Italy by scholars and astronomers, and that in fact the prohibition would increase their influence. Kepler himself addressed an open letter to the foreign booksellers, especially those in Italy, asking that the censors should examine the new evidence he had produced in favor of Copernicus, unfortunately too late to prevent the prohibition. In the meantime he requested the booksellers to restrict the sales of his *Harmonice mundi* to the highest clergy and most eminent mathematicians and philosophers (KGW 6, pp. 543–544).

if not in that vast and immense space within the sphere of the fixed stars, yet at least in the system of the planetary world, takes the same position at any given time according to Brahe as Copernicus gives it. In fact, just as someone who draws a circle on paper moves the writing leg of his compasses round, whereas someone who fastens his paper or tablet to a revolving wheel describes the same circle, without moving the leg of his compasses or his pen, on the tablet as it moves round; in the same way in this case for Copernicus indeed the Earth traces out a circle by a real motion of its own body, passing in between the circles of Mars on the outside and Venus on the inside; but for Tycho Brahe the whole planetary system (in which among the others are also the circles of Mars and Venus) turns, like the tablet on the wheel, applying to the motionless Earth, as if to the pen of the man who turns the wheel, the blank space between the circles of Mars and Venus.[33] The effect of this motion of the system is that the Earth marks on it the same circle round the Sun, intermediate between those of Mars and Venus, while itself it remains motionless, as according to Copernicus it marks out by a true motion of its own body, with the system at rest. Therefore, as harmonic study considers the motions of the planets as eccentric, as if viewed from the Sun,[34] it may readily be understood that if an observer were on the Sun, even though it were in motion, to him the Earth, although it were at rest (to make a concession already to Brahe), would nevertheless appear to be going around an annual circuit, placed in between the planets, and also in an intermediate period of time. Hence if there is a man whose confidence is too weak for him to be able to accept the motion of the Earth among the stars, nevertheless he will be able to rejoice in the marvelous study of this absolutely divine mechanism, if he applies whatever he is told about the daily motions of the Earth on its eccentric to their appearance from the Sun, the same appearance as Tycho Brahe shows, with the Earth at rest.

Harmonic study valid in the hypotheses of Brahe.

Much more valid in the Copernican ones.

However, true enthusiasts for the Samian philosophy[35] have no just cause to grudge such people this share in a most delightful speculation,

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[33] The analogy is not exact, for in the case of the rotating wheel both Earth and Sun would be at rest, whereas the Sun should be allowed to circulate about the Earth.

[34] Kepler seeks the celestial harmonies in the motions of the planets as seen from the sun, because he believes these to be the true motions; that is, the actual motions free from the distortion of combination with the earth's motion. The same harmonies would be found in the Tychonic system, because the motions as seen from the sun were in fact the true motions, even if followers of Tycho would not have regarded them as such. Ptolemy, however, had only the apparent motions as seen from the earth, and for this reason it was impossible for him to have discovered the celestial harmonies. In order to avoid confusion, it should be noted that Kepler also uses the expression "true motion" to refer to the angular motion as seen from the center of the eccentric. The harmonies are based on the appearance of this true motion as seen from the sun, that is, the apparent angular velocity with respect to the sun.

[35] That is, the heliocentricism of Aristarchus of Samos, to whom the first statement of the Copernican hypothesis has been attributed. See T. Heath (1981).

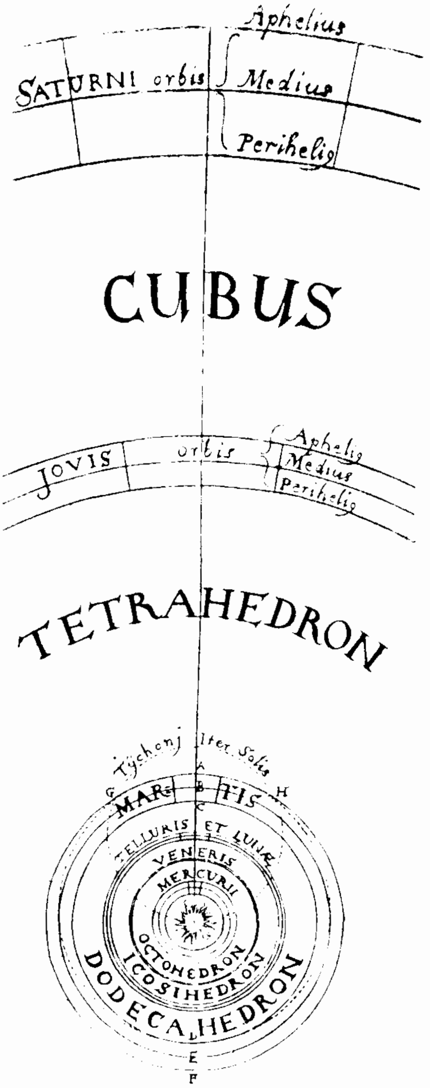
inasmuch as their joy will be many times more perfect, that is from the complete perfection of the speculation, if they do also accept the immobility of the Sun and furthermore the movement of the Earth.

I. The bodies of which the circuits establish the harmonies.

First, therefore, readers should take it as absolutely settled today among all astronomers that all the planets go round the Sun, with the exception of the moon, which alone has the Earth as its center; and its orbit or course is not large enough to be capable of being drawn in its correct proportion to the rest in the following plan. Therefore the Earth is added to the other five as a sixth, which either by its own motion, with the Sun at rest, or without moving itself while the whole system of the planets revolves, itself also marks out a sixth circle round the Sun.

II. The variation in the separation of these bodies.

Second, it is also settled that all the planets are eccentric, that is, they change their distances from the Sun, in such a way that on one side their orbits are furthest from the Sun, in the other they come closest to the Sun. In the attached diagram three circles have been constructed for each of the planets. None of them indicates the actual eccentric path of the planet, but the middle one in fact, for instance *BE* in the case of Mars, is equivalent to the eccentric orbit, with respect to its longer diameter; but the actual orbit, for instance *AD*, touches the upper of the three, *AF*, on one side *A*, and the lower *CD* on the opposite side *D*. The circle *GH* which is sketched out by dots, and drawn through the center of the Sun, indicates the path of the Sun according to Tycho Brahe. If it moves on this path absolutely all the points in the whole planetary system here depicted proceed on an equivalent path, each on its own. And if one point on it, that is to say the center of the Sun, stops in one part of its circle, as here at the lowest point, then absolutely all points of the system will stop, each at the lowest parts of their own circles. Also the three circles of Venus on account of the restricted space have merged into one, contrary to my intention.



Orbit of SATURN at { *aphelian* / *mean* / *perihelian*

CUBE

Orbit of JUPITER at { *aphelian* / *mean* / *perihelian*

TETRAHEDRON

Tychonic path of Sun
A
G *B* *H*
of MARS *C*
of Earth and Moon
of VENUS
of MERCURY

OCTOHEDRON
ICOSIHEDRON
DODECAHEDRON
D
E
F

III. The reason for the number of the bodies which make the harmonies.

Third, the reader should remember what I published in *The Secret of the Universe*, 22 years ago, that the number of the planets, or of courses round the Sun, was taken by the most wise Creator from the five regular solid figures, about which Euclid so many centuries ago wrote a book which is called the *Elements*, on account of its being made up of a series of Propositions. However, the fact that there cannot be more regular solids, that is, that regular plane figures cannot be congruent in a solid in more than five ways, has been made clear in Book II of this work.

IV. The reason for the size of the spheres which pass between the solids.

Fourth, as far as the proportion of the planetary orbits is concerned, between pairs of neighboring orbits indeed it is always such as to make it readily apparent that in each case the proportion is close to the unique proportion of the spheres of one of the solid figures, that is to say the proportion of the circumscribed sphere of the figures to the inscribed sphere. However, it is not definitely equal, as I once dared to promise for eventually perfected astronomy. For after the final verification of the intervals, from the observations of Brahe, I discovered the following facts: if the vertices of the cube are indeed applied to the inside circle of Saturn, the centers of the faces almost touch the middle circle of Jupiter; and if the vertices of the tetrahedron rest on the inside circle of Jupiter, the centers of the faces of the tetrahedron almost touch the outside circle of Mars.[36] In the same way the vertices of the octahedron, which rise from any of the circles of Venus (as they are all three compressed into a very narrow gap) are penetrated by the centers of the faces of the octahedron, which go down more deeply below the outside circle of Mercury, yet do not reach as far as the middle circle of Mercury. Finally, the closest of all to the proportions of the dodecahedric and icosahedric spheres, which are equal to each other, are the proportions or intervals between the circles of Mars and the Earth, and between those of the earth and Venus, which are similarly equal to each other, if we reckon from the inside circle of Mars to the middle circle of the Earth, but from the middle circle of the Earth to the middle circle of Venus. For the mean distance of the Earth is the mean proportional between the smallest circle of Mars and the middle one of Venus. However, these two proportions between the circles of the planets are still greater than are those of the pairs of circles in the figures belonging to the spheres, to the extent that the centers of the faces of the dodecahedron do not touch the outer circle of the Earth, nor the centers of the faces of the icosahedron the outer circle of Venus. Yet this gap is not filled up by

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[36] For a perfect fit, the circumscribed sphere should correspond to the perigee of the outer planet and the inscribed sphere to the apogee of the inner planet. Thus the tetrahedron fits perfectly in the space between Jupiter and Mars. The spouses cube and octahedron both penetrate their planetary spheres. On the other hand, the spouses dodecahedron and icosahedron leave a gap. Kepler's attempt to rationalize the discrepancies is described in note 28.

adding the semidiameter of the moon's orbit above the greatest interval of the Earth and subtracting it from below the smallest interval. However, I discover another proportion in the figures: namely that if the augmented dodecahedron to which I have given the name of Echinus (Hedgehog), that is to say the one formed from twelve quinquagonal stars, and therefore very close to the five regular solids, if, I say, it places its twelve points on the inner circle of Mars, then the sides of the pentagons which are individually the bases of the rays or points touch the middle circle of Venus.[37] In brief, the cube and octahedron which are spouses do penetrate their planetary spheres somewhat; the dodecahedron and icosahedron which are spouses do not altogether follow theirs, whereas the tetrahedron exactly touches both. In the first case there is a deficiency, in the second an excess, and in the last an equality in the intervals of the planets. Page 396.

From that fact it is evident that the actual proportions of the planetary distances from the Sun have not been taken from the regular figures alone; for the Creator, the actual fount of geometry, who, as Plato wrote, practices eternal geometry, does not stray from his own archetype.[38] And that could certainly be inferred from the very fact that all the planets change their intervals over definite periods of time, in such a way that each one of them has two distinctive distances from the Sun, its greatest and its least; and comparison of distances from the Sun between pairs of planets is possible in four ways, either of the greatest distances, or of the least, or of the distances on opposite sides when they are furthest from each other, or when they are closest. Thus the comparisons between pair and pair of neighboring planets are twenty in number, whereas on the other hand there are only five solid figures. However, it is fitting that the Creator, if He paid attention to the proportion of the orbits in general, also paid attention to the proportion between the varying distances of the individual orbits in particular, and that that attention should be the same in each case, and that one should be linked with another. On careful consideration, we shall plainly reach the following conclusion, that for establishing both the diameters and the eccentricities of the orbits in conjunction, more basic principles are needed in addition to the five regular solids.

V. Astronomical axioms of the true motions.

Fifth, to come to the motions, between which harmonies are established, I again impress on the reader that it was shown by me in my *Commentaries on Mars*, from the thoroughly reliable observations

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[37] The echinus fills the gap between Mars and Venus. This derivative solid thus provides an explanation of the actual magnitude of the discrepancy between the planetary distances and the interpolation of the dodecahedron and icosahedron. Although the requirements of the celestial harmony necessitated an adjustment of the distances as indicated by the interpolation of these polyhedra, the geometrical archetype nevertheless included an element from which the required deviations could be derived *a priori*.

[38] The expression is not actually to be found in Plato's writings. See Plutarch, *Convivia*, viii, 2.

of Brahe, that equal daily arcs on one and the same eccentric are not traversed at the same speed;

1. but that these differing *times expended on equal parts of the eccentric observe the proportion of their own distances from the Sun*, the fount of motion;[39] and in turn, that supposing equal times, say one natural day in each case,
2. *the true daily arcs of a single eccentric orbit* corresponding with them *have a proportion to each other which is the inverse of the proportion of the two distances from the Sun.*
3. At the same time, however, it was shown by me that *the orbit of a planet is elliptical,*
4. *and the Sun*, the fount of motion, *is at one of the focuses of that ellipse*;
5. and thus it comes about that the planet, when it has completed out of its whole circuit *a quadrant from its aphelion, is at precisely its mean distance from the Sun*, between its greatest at aphelion and its least at perihelion.
6. From these two axioms the conclusion is drawn that the daily *mean motion* of the planet on its eccentric *is the same as the true* daily arc of its eccentric, at those moments at which the planet is *at the end of the quadrant of its eccentric as reckoned from the aphelion*, even though that true quadrant still appears smaller than a proper quadrant.
7. Furthermore, it follows that *any two really true daily arcs of the eccentric, which are truly at equal distances, one from the aphelion and the other from the perihelion, added together are equal to two mean daily arcs*;[40]
8. and in consequence, since the proportion of the circles is the same as that of their diameters, that *the proportion of one mean daily arc to the sum of all the mean daily arcs* making up the whole circuit, which are equal to each other, *is the same as that of a mean daily arc to the sum of all the true* eccentric arcs, which are the same in number but unequal to each other. We need to have this knowledge of the true eccentric daily arcs, and of the true motions, beforehand so that we can now grasp through them the apparent motions, supposing the eye to be at the Sun.

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[39] Kepler here alludes to his distance law, based on his physical explanation of planetary motion, according to which the velocity of a planet in its orbit is inversely proportional to its distance from the sun. On the relation of Kepler's distance and area laws, see E.J. Aiton (1969).

[40] This is only approximately true but for small eccentricities the error is insignificant. According to the distance law, the sum of the two arcs s_1 and s_2 (with corresponding distances r_1 and r_2) is $(2r^2s)/(r_1r_2)$, where s and r are the mean arc and distance respectively. The sum of the two arcs would be $2s$ if r were taken to be the geometric mean of r_1 and r_2 instead of the arithmetic mean.

VI. Astronomical axioms of the apparent motions or angles as seen from the Sun, which are included daily between lines originating from the center of the Sun.

Sixth, as far indeed as concerns the apparent arcs as seen from the Sun, it has been known even from the time of ancient astronomy that of the true motions, even those which are equal to each other, one which has moved further away from the center of the world (such as one which is at *aphelion*) *seems to be smaller*, to an observer at that center; and one which is nearer, such as *one which is at perihelion, also seems to be greater.* Since therefore in addition the true daily arcs are also greater when close, on account of their motion's being faster, but lesser when at a distance at aphelion on account of the slowness of the motion,

2. hence I have shown in the *Commentaries on Mars* that *the proportion of the apparent daily arcs on a given eccentric is fairly precisely the square of the inverse proportion of their distances from the Sun.*[11]
   Thus if a planet on a particular one of its days, when it is at aphelion, were at a distance of 10 parts from the Sun, in any units, and on the opposite day, when it is at perihelion, at a distance of 9 parts, in similar units, it is certain that at aphelion its apparent forward motion as seen from the Sun will be to its apparent motion at perihelion as 81 to 100.
3. However, that is true with the following reservations; first, *that the eccentric arcs are not large*, so that they do not participate in different distances which vary considerably, that is so that they do not produce an appreciable difference in the distance of their ends from the apsides;
4. second, *that the eccentricity is not very large*, for the larger the eccentricity, that is to say the larger the arc is, the more the angle which it appears to subtend is increased, beyond the bound set by its closeness to the Sun, *by Theorem 8 of Euclid's Optics.*[12]
5. However, it *is of no importance in small arcs and at a great distance*, as I have commented in my *Optics*, Chapter XI. But there is another reason for me to comment on this point.
6. For *arcs of the eccentric near the mean anomalies are viewed obliquely from the center of the Sun*, and this obliquity diminishes the apparent size,
7. whereas on the contrary *arcs near the apsides present themselves*

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[11] Let the daily motions (that is, the apparent arcs) at aphelion and in the mean distances be M_a and M. Then, if S_a and S are the corresponding true arcs, and R_a and R the corresponding distances, $S_a/S = (M_a/M)(R_a/R)$. Owing to the weakening of the solar force, $S_a/S = R/R_a$. Hence $M_a/M = R^2/R_a^2$. Similarly $M/M_p = R_p^2/R^2$, where M_p and R_p are the daily motion and distance respectively at perihelion. Combining the two results, it follows that $M_a/M_p = R_p^2/R_a^2$. See *Astronomia nova*, Chapter 32.

[12] *Euclides optica et catoptrica . . . eadem Latine reddita per I. Penam* (Paris, 1557). Cf. the French translation by Paul ver Eecke, *Euclide, L'Optique et la catoptrique* (Paris/Bruges, 1938), pp. 6–7.

normally to an observer, so to speak, placed on the Sun.[13] Therefore, when the eccentricity is very large, the sensible damage is done to the proportion of the motions, if we apply the mean diurnal motion undiminished to the average distance, as if it appeared from the average distance to be the size which it actually is, as will be apparent below in the case of Mercury. All this is related at greater length in the *Epitome of Copernican Astronomy*, Book V. Nevertheless it had to be recalled here as well, because it concerns the actual terms of the heavenly harmonies, considered separately on their own.

VII. Rejection of the motions which are apparent to observers on Earth.

Seventh, if anyone should bring to mind the daily motions not as they appear to observers from the Sun, so to speak, but from the Earth, with which Book VI of the *Epitome of Copernican Astronomy* deals, he should know that no account of them whatever is taken in this proceeding, and definitely none should be. For the Earth is not the fount of their motions, and cannot be, for those motions degenerate not only into mere rest or apparent standstill, but into definite retrogression, as far as the deceptive appearance is concerned. On that basis all the infinity of proportions is attributable to all the planets simultaneously and equally. Therefore, to make certain what proportions are established as their own by the daily motions of the true eccentric orbits individually (even though they are themselves still apparent, to an observer, so to speak, on the Sun, the fount of motion), this fantasy, common to all five, of an adventitious annual motion must be removed from those proper motions, whether it arises from the motion of the Earth itself, according to Copernicus, or from the annual motion of the whole system, according to Tycho Brahe; and the motions proper to each planet, stripped of inessentials, must be brought into view.[14]

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[13] This difference played an important role in Kepler's definitive clarification of the area law, when he demonstrated that his original inverse-distance law had to be understood to mean that the trans-radial velocity of the planet was inversely proportional to the distance and not the velocity in the orbit itself. In this formulation, the inverse-distance law is equivalent to the area law. See Aiton (1969), 87–88.

[14] Kepler here reiterates his view that the celestial harmonies are to be sought in the true motions of the planets, which are those that would be seen from the sun. It would be possible to recognize the harmonies in the Tychonic system, because this differs from the Copernican only by the addition of an illusory annual motion of the whole system, which is easily abstracted. From the fact that the celestial harmonies would only be perceived from the sun, it would seem to follow that Kepler regarded them as objects of the mind and not the senses. While the earth-soul could perceive and be influenced by the astrological aspects (the manifestation of harmony in nature), it would seem that only intelligent minds could understand and recognize the celestial harmonies, once they had been brought to light. And as Kepler remarked in his introduction to Book V, God had waited six thousand years for this to happen. However, in Chapter IV, he played with the idea of an instinctive recognition of the celestial harmonies, conveyed in some way which he does not specify, along with rays of light from the Sun. Cf. note 59.

Eighth, up till now we have dealt with the various elapsed times or arcs of one and the same planet. Now we must also deal with the motions of pairs of planets compared with each other. Here note the definitions of the terms which we are going to need. We shall call the *nearest apsides* of two planets the perihelion of the upper one and the aphelion of the lower one, notwithstanding the fact that they are tending not to the same side of the world, but to different, and perhaps opposite sides.

VIII. What is the proportion of the periodic times to the distances from the Sun of any pair of planets?

2. By *extreme motions* understand the slowest and the fastest of the whole planetary circuit.
3. By *converging or approaching motions*, those which are at the nearest apsides of the two, that is at the perihelion of the upper planet and the aphelion of the lower;
4. by *diverging or receding motions*, those which are at opposite apsides, that is at the aphelion of the upper planet and the perihelion of the lower. Again, therefore, a part of my *Secret of the Universe*, put in suspense 22 years ago because it was not yet clear, is to be completed here, and brought in at this point. For when the true distances between the spheres were found, through the observations of Brahe, by continuous toil for a very long time, at last, at last, the genuine proportion of the periodic times to the proportion of the spheres—

   only at long last did she look back at him as he lay motionless,
   But she looked back and after a long time she came;[15]

   and if you want the exact moment in time, it was conceived mentally on the 8th March in this year one thousand six hundred and eighteen, but submitted to calculation in an unlucky way, and therefore rejected as false, and finally returning on the 15th of May and adopting a new line of attack, stormed the darkness of my mind. So strong was the support from the combination of my labor of seventeen years on the observations of Brahe and the present study, which conspired together, that at first I believed I was dreaming, and assuming my conclusion among my basic premises. But it is absolutely certain and exact that *the proportion between the periodic times of any two planets is precisely the sesquialterate proportion of their mean distances, that is, of the actual spheres,*[16] though with this in mind, that *the arithmetic mean*

For in the *Commentaries on Mars*, Ch. XLVIII, p. 232, I have proved that *this arithmetic mean is either the actual diameter of the circle which is equal in length to the elliptical orbit or slightly smaller.*

[15] Vergil, *Eclogue* I, 27 and 29.

[16] In the *Mysterium cosmographicum*, physical considerations led Kepler to a relation between the periodic times and the mean distances expressed by the formula $(T_2 - T_1)/T_1 = 2(r_2 - r_1)/r_1$, where T_1, T_2 represent the periodic times and r_1, r_2 the mean distances and r_2 is greater than r_1. See Duncan (1981), pp. 201 and 249. In the *Astronomia nova* (Chapter 39), this was modified to $T_1 : T_2 = r_1^2 : r_2^2$. As he here relates, he discovered the correct law, $T_1^2 : T_2^2 = r_1^3 : r_2^3$, on 15 May 1618. An *a priori* (physical) explanation of the law was first given in the third part of the *Epitome astronomiae*

between the two diameters of the elliptical orbit is a little less than the longer diameter. Thus if one takes *one third of the proportion from the period*, for example, of the Earth, which is one year, and the same from *the period* of Saturn, thirty years, that is, *the cube roots*, and one doubles that proportion, *by squaring* the roots, *he has in the resulting numbers the exactly correct proportion of the mean distances* of the Earth and Saturn *from the Sun.* For the cube root of 1 is 1, and the square of that is 1. Also the cube root of 30 is greater than 3, and therefore the square of that is greater than 9. And Saturn at its average distance from the Sun is a little higher than nine times the average distance of the Earth from the Sun. The use of this theorem will be necessary in Chapter IX for the derivation of the eccentricities.

IX. How large a space any planet traverses relatively to another in any given time.

Ninth, if you want to measure the actual completely true daily paths of each planet through the aethereal air, with, so to speak, a ten foot rule, two proportions will have to be combined, one that of the true (not the apparent) daily arcs of the eccentric, the other that of the average distances of each planet from the Sun, because it is the same as that of the width of the orbits. *That is, the true daily arc of each planet must be multiplied by the semidiameter of its orbit.* When that has been done, the resulting figures will be convenient for investigating whether those paths make harmonic proportions.

X. How from the true paths, and the true distances of the planets from the Sun, is found the apparent motion as from the Sun, the subject of celestial harmony.

Tenth, to find definitely the apparent size of any such daily path, to an eye placed, so to speak, on the Sun, although the same thing can immediately be sought through astronomy, yet it will also be revealed if you multiply the proportion of the paths by the inverse proportion not of the mean, but of the true distances as they are at any point of the eccentrics: *by multiplying the path of the upper by the distance of the lower from the Sun, and correspondingly the path of the lower by the distance of the upper from the Sun.*

XI. How from the apparent diurnal motions (seen, so to speak, from the Sun), are elicited the distances of the planets from the Sun.

Eleventh, in the same way also from given apparent motions, at aphelion for one planet and at perihelion for the second, or the other way round, the proportions of the distances are elicited, of one at aphelion to that of the second at perihelion. In this case, however, the mean motions must be known in advance, that is the inverse pro-

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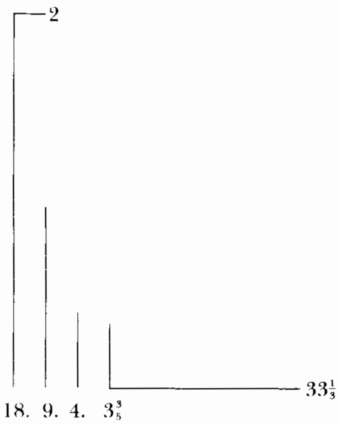
Copernicanae (KGW 7, p. 307), published in 1621. There he explains that the periodic times depend on four factors. Of these, the length of the orbital path (proportional to the mean distance r) and the strength of the solar force (inversely proportional to r) would by themselves combine to give the relation described in the *Astronomia nova.* The other two factors are the mass of the planet, which on the basis of archetypal reasons he took to be proportional to $\sqrt{r}$, and the volume (measuring the ability of the planet to assimilate the solar force), which on the basis of observational evidence he felt he could take proportional to r. In accordance with his Aristotelian dynamics, where speed was proportional to force and inversely proportional to resistance (represented by the mass), the four factors combined to give the third or harmonic law.

portion of the periodic times, from which the proportion of the orbits is elicited, by Number VIII stated above: then *by taking the mean proportional between one or the other apparent motion and their mean, it turns out that this mean proportional is to the semidiameter*, which has already been revealed, of the *orbit, as is the mean motion to the separation or distance, which is required.* Let the periodic times of two planets be 27 and 8. Then the proportion of the mean daily motion of the former to the latter is as 8 to 27. Hence the semidiameters of the orbits will be as 9 to 4. For the cube root of 27 is 3; that of 8 is 2; and the squares of these roots are 9 and 4. Now let the apparent motion at aphelion of one be 2, and at perihelion of the other 33 and a third. The mean proportionals between the mean motions 8 and 27, and these apparent motions, will be 4 and 30. Therefore, if the mean 4 gives an average distance for the planet of 9, then a mean motion of 8 yields a distance at aphelion of 18, corresponding with an apparent motion of 2. And if the other mean, 30, gives an average distance for the other planet of 4, then its mean motion of 27 gives its distance at perihelion as $3\frac{3}{5}$. Therefore, I say that its distance at aphelion is to its distance at perihelion as 18 to $3\frac{3}{5}$. From that it is evident that the harmonies dictated between the extreme motions of the two, and the periodic times prescribed in each case, entail the extreme and average distances, and so also the eccentricities.[17]



Twelfth, from the receding extreme motions of one and the same planet it is possible to find *the mean motion. For in this case it is not precisely the arithmetic mean between the extreme motions, nor precisely the geometric mean; but it is less than the geometric mean by the same amount as the geometric mean is less than the mean between the two.*[18] Let there be two

XII. The proportion of the mean motion to the extreme motions.

[17] The calculation is easier to follow when formulated in modern algebraic notation. Let R_p, r_p (where R_p is greater than r_p) be the distances of the two planets at perihelion, R_a, r_a the corresponding distances at aphelion, and R, r the respective mean distances. Also let M_p, m_p be the motions at perihelion, M_a, m_a the motions at aphelion and M, m the mean motions. Given M_a/m_p and M/m, Kepler calculates r_p/R_a. As the mean daily motions are inversely proportional to the periodic times, the third (harmonic) law gives (1) $m/M = (R/r)^{3/2}$. From Number 6 of this chapter (see note 41), it follows that (2) $M_a/M = R^2/R_a^2$ and $m_p/m = r^2/r_p^2$. For a given ratio of periodic times, namely 27:8, Kepler takes the mean distances as $R = 9$, $r = 4$. The corresponding values of the motions are $M = 8$, $m = 27$. Then taking $M_a = 2$, $m_p = 33\frac{1}{3}$, he calculates the auxiliary quantities $M_1 = \sqrt{M_a M} = 4$ and $m_1 = \sqrt{m_p m} = 30$. Using (2), $R_a = MR/M_1 = 18$ and $r_p = mr/m_1 = 3\frac{3}{5}$. Hence $r_p:R_a = 1:5$.

[18] Using the notation of the previous note for the inner planet, and in addition taking $G = \sqrt{m_a m_p}$ and $A = \frac{1}{2}(m_a + m_p)$, Kepler's formula becomes $m = G - \frac{1}{2}(A -$

extreme motions, 8 and 10. The mean motion will be less than 9 and also less than the square root of 80 by half the difference between the two, that is between 9 and the square root of 80. Thus if the motion at aphelion is 20 and at perihelion 24, the mean motion will be less than 22, and also less than the square root of 480 by half the difference between that root and 22. The application of this theorem is in what follows.

XIII. The relationship of the proportion of the distances between the two planets and the Sun to the proportion of the apparent motions of each of them.

Thirteenth, from what has already been stated is proved the proposition, which will be very necessary to us, that as the proportion of the mean motions in the two planets, so is the inverse of the square root of the cube of the proportion of the orbits. Thus *the proportion* of two *apparent converging extreme motions is always less than the sesquialterate of the proportion*[49] *of the distances corresponding with those* extreme motions; *and by the same amount as, multiplied together, the* two *proportions of two corresponding distances to the* two *mean distances or to the semidiameters of the* two *orbits come to less than the square root of the proportion of the orbits, the proportion of* the two extreme *converging motions is greater than the proportion of the corresponding distances; whereas if that product exceeded the square root of the proportion of the orbits, then the proportion of the converging motions would be less than the proportion of their distances.*[50]

Let the proportion of the orbits be *DH:AE*, and the proportion of the mean motions *HI:EM*, the sesquialterate of the inverse of the former. Let the distance of the orbit, that is *CG*, be at its smallest in

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G). Although Kepler offers no justification, the result may be established as follows. Multiplying the relations $\frac{m}{m_a} = \frac{r_a^2}{r^2}$ and $\frac{m}{m_p} = \frac{r_p^2}{r^2}$ gives $\frac{r_a r_p}{r^2} = \frac{m}{G}$.

Then adding the same relations gives

$$\frac{m(m_a + m_p)}{m_a m_p} = \frac{r_a^2 + r_p^2}{r^2} \quad \text{or} \quad \frac{2mA}{G^2} = \frac{(r_a + r_p)^2 - 2r_a r_p}{r^2}; \quad \text{that is } \frac{mA}{G^2} = 2 - \frac{m}{G}.$$

Hence

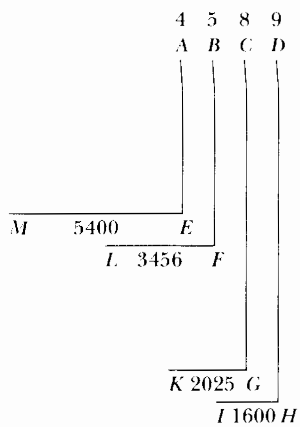
$$m = \frac{2G^2}{A + G} = G\left(1 + \frac{A - G}{2G}\right)^{-1} = G - \tfrac{1}{2}(A - G),$$

neglecting the square and higher powers of $(A - G)$.

[49] This means the inverse proportion.

[50] First, it needs to be observed that Kepler takes a proportion to be greater according to the difference of the quotient of the numbers forming the proportion from 1. Thus the first part of Kepler's proposition—the proportion of two apparent converging extreme motions is always less than the 3/2th power of the inverse proportion of the distances—would be formulated in modern notation as $M_p/m_a > (r_a/R_p)^{3/2}$, since the quotients are less than 1. Similarly, the second part of the proposition may be formulated $M_p/m_a < r_a/R_p$ when $(rR_p)/(Rr_a) > (r/R)^{1/2}$ (that is, when $r_a/R_p < (r/R)^{1/2}$), and $M_p/m_a > r_a/R_p$ when $(rR_p)/(Rr_a) < (r/R)^{1/2}$ (that is, when $r_a/R_p > (r/R)^{1/2}$. From $M_p/M = R^2/R_p^2$ and $m_a/m = r^2/r_a^2$, it follows that $(M_p m)/(M m_a) = (R^2 r_a^2)/(r^2 Rp^2)$. Using the harmonic law $M/m = (r/R)^{3/2}$, this becomes $M_p/m_a = (R/r)^{1/2} \cdot (r_a^2/R_p^2)$. Since $R/r > R_p/r_a$ and therefore $(R/r)^{1/2} > (R_p/r_a)^{1/2}$, it follows that $M_p/m_a > (r_a/R_p)^{3/2}$. If now $r_a/R_p < (r/R)^{1/2}$, it follows from $M_p/m_a = (R/r)^{1/2} \cdot (r_a^2/R_p^2)$ that $M_p/m_a < r_a/R_p$, and if $r_a/R_p > (r/R)^{1/2}$, that $M_p/m_a > r_a/R_p$.

the former case, and of the orbit in the latter case, that is *BF*, at its greatest; and let the product of the proportions *DH:CG* and *BF:AE* be in the first instance less than the square root of DG:*AE*. Also let the apparent motion of the upper planet at perihelion be *GK*, and of the lower at aphelion *FL*, so that they are extreme converging motions. I say that the proportion *GK:FL* is greater than the inverse of the proportion *CG:BF*, but less than its sesquialterate. For the proportion of *HI* to *GK* is the square of the proportion of *CG* to *DH*; and the proportion of *FL* to *EM* is the square of the proportion of *AE* to *BF*. Therefore, the two proportions multiplied together, that of *HI* to *GK* and of *FL* to *EM*, come to the square of the proportions of *CG* to *DH* and of *AE* to *BF* multiplied together. But the proportions of *CG* to *DH* and of *AE* to *BF* multiplied together are less than the square root of the proportion of *AE* to *DH* by a definite amount, as in the assumptions.



Therefore, the proportions of *HI* to *GK* and of *FL* to *EM* multiplied together are also less than the square of the square root, that is, less than the whole proportion of *AE* to *DH*, by a factor which is the square of the previous deficiency. But *HI* to *EM* is the sesquialterate of the proportion of *AE* to *DH*, by VIII previously stated. Then less than the square of the deficiency divided into the sesquialterate of the proportion, or in other words the proportions of *HI* to *GK* and of *FL* to *EM* divided into the proportion of *HI* to *EM* leave as quotient more than the square root of the proportion of *AE* to *DH*, by the square of the amount in excess. But they yield as quotient the proportion of *GK* to *FL*. Therefore, the proportion of *GK* to *FL* is more than the square root of the proportion of *AE* to *DH* by the square of the factor in excess. But the proportion of *AE* to *DH* is made up of three proportions, those of *AE* to *BF*, of *BF* to *CG*, and of *CG* to *DH*. Also the proportion of *CG* to *DH* together with that of *AE* to *BF* is less than the square root of that of *AE* to *DH*, by a deficiency of the simple factor. Therefore, the proportion of *BF* to *CG* is more than the square root of that of *AE* to *DH*, by the simple factor. But the proportion of *GK* to *FL* was also more than the square root of that of *AE* to *DH*, in fact by the square of the excess factor. However, the square of the excess is greater than the simple factor. Therefore, the proportion of the motions *GK* to *FL* is greater than the proportion of the corresponding distances, *BF* to *CG*.

Clearly it is shown in the same way that in the opposite case, if the planets come close to each other at *G* and *F*, beyond the mean separations at *H* and *E*, in such a way that the proportion of the mean separations *DH*, *AE* loses more than its square root, then the proportion of the motions *GK* to *FL* becomes less than the proportion of their distances, *BF* to *CG*. For nothing more needs to be done than to change the words *greater* to *lesser*, *more* to *less*, *excess factor* to *deficiency*, and the other way round.

In the numbers quoted, the square root of 4:9 is 2:3, and 5:8 is still greater than 2:3 by a factor of 15:16 in excess. Also the proportion 8:9 squared is the proportion 1600:2025, that is 64:81; and the proportion 4:5 squared is the proportion 3456:5400, that is 16:25; and lastly the sesquialterate of the proportion 4:9 is the proportion 1600:5400, that is 8:27. Therefore, also the proportion 2025 to 3456, that is 75:128, is still greater than 5:8, that is 75:120, by an excess factor of the same amount (120:128, that is), 15:16. Hence *the proportion of the converging motions, 2025:3456, exceeds the inverse proportion of the corresponding distances, 5:8, by the same factor as the latter exceeds the square root of the proportion of the orbits, 4:9.* Or, which comes to the same thing,[51] *the proportion of the two converging distances is the mean between the square root of the proportion of the orbits and the inverse proportion of the corresponding motions.*

From that, however, we may infer that *the proportion of the diverging motions is much greater than the sesquialterate of the proportion of the orbits*, since the sesquialterate is multiplied by the square of the proportions of the distance at aphelion to the mean distance, and of the mean distance to that at perihelion.

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[51] For his numerical example, Kepler takes $DH = R = 9$, $AE = r = 4$, $CG = R_p = 8$, $BF = r_a = 5$, $HJ = M = 1600$, $EM = m = 5400$, $GK = M_p = 2025$ and $FL = m_a = 3456$. Then $r/R = 4/9$, so that $(r/R)^{1/2} = 2/3$, while $r_a/R_p = 5/8$. Hence the proportion $r_a{:}R_p$ is greater (in Kepler's terms) that the proportion $r{:}R$, the excess being the proportion 15:16, obtained by dividing 2/3 into 5/8. Again $(R_p/R)^2 = 64/81 = 1600/2025 = M/M_p$ and $(r/r_a)^2 = 16/25 = 3456/5400 = m_a/m$. Also $(r/R)^{3/2} = 8/27 = 1600/5400 = M/m$. Hence the proportion $M_p{:}m_a = 2025{:}3456 = 75{:}128$ is greater (in Kepler's terms) than the proportion $r_a{:}R_p = 5{:}8$, the excess being 15:16, obtained by dividing 5/8 into 75/128. Thus the excess of $M_p{:}m_a$ over $r_a{:}R_p$ equals the excess of $r_a{:}R_p$ over $r^{1/2}{:}R^{1/2}$.

Chapter IV.

In What Features Relating to the Motions of the Planets Have the Harmonic Proportions been Expressed by the Creator, and How?

When therefore the fantasy of retrogressions and stations has disappeared, and the planets' proper motions, in their own true eccentric orbits, have been stripped to essentials, there still remain in the planets the following distinct features: 1. their distances from the Sun; 2. their periodic times; 3. their daily eccentric arcs; 4. the daily times expended on their arcs; 5. their angles at the Sun, or apparent daily arcs to observers, so to speak, on the Sun. And again, all of these (except for the periodic times) are variable right around their orbit, most indeed at the mean longitudes, and least in fact at the extremities, when they have just turned away from one of them and are returning towards the opposite one. Hence when the planet is lowest and closest to the Sun, and therefore expends as little time as possible on one degree of its eccentric, and on the other hand completes its greatest daily arc of the eccentric in a single day, and appears fastest from the Sun, then its motion persists for a while in this vigorous state, without sensible variation, until when the perihelion has been passed the planet has begun to increase its linear distance from the Sun. Then at the same time it also expends a longer time on the degrees of its eccentric, or if you consider the motion of a single day, it makes less progress on each following day, and also appears much slower from the Sun, until it approaches its upper apsis, making its distance from the Sun the greatest. For then it also expends the longest time of all on one degree of its eccentric, or on the other hand completes its smallest arc in one day, and also makes its appearance much smaller and the smallest in its whole circuit.

Lastly, all these features belong either to any one planet at different times, or to different planets; so that if we suppose an infinity of time, all the states of the orbit of one planet can coincide at the same moment of time with all the states of the orbit of another planet, and can be compared; and then the complete eccentrics indeed, compared with each other, have the same proportion as their semidiameters, or their average distances, whereas the arcs of the two eccentrics, designated as equal or by the same number, nevertheless have unequal true lengths in the proportion of the whole eccentrics. For example, one degree on the sphere of Saturn is almost twice as large as one degree on the sphere of Jupiter. And on the other hand, the daily arcs of the eccentrics, expressed in astronomical numbers, do not show the same proportion as the true paths, which the globes complete through the aethereal air in one day because single degrees each represent on the wider circle of the superior planet a section of its path which is larger, but on the narrower circle of the inferior planet a section which is smaller. Hence a sixth aspect for consideration is now added, concerning the daily paths of the two planets.

First, therefore, let us take the second of the features listed, that is to say the periodic times of the planets, which comprise the assembled totals of all the times expended on all the degrees of the whole circuit, long, average, and small.[52] And it has been observed from antiquity up to our own time that the planets complete their journeys around the Sun as follows in the table.

	Days	*Sixtieths of a day*	*Therefore mean daily motions:* *Minutes*	*Seconds*	*Third minutes*
Saturn	10759	12	2	0	27
Jupiter	4332	37	4	59	8
Mars	686	59	31	26	31
Earth with Moon	365	15	59	8	11
Venus	224	42	96	7	39
Mercury	87	58	245	32	25

In these periodic times there are therefore no harmonic proportions, which is readily apparent if the larger periods are continually divided by two, and the smaller ones are continually doubled, so that with the intervals of a diapason suppressed we can look for those which are within a single diapason.[53]

	Saturn	*Jupiter*	*Mars*	*Earth*	*Venus*	*Mercury*	
	10759. 12						
Halves	5379. 36	4332. 37				87. 58	Doubles
	2689. 48	2166. 19			224. 42	175. 56	
	1344. 54	1083. 10	686. 59	365. 15	449. 24	351. 52	
	672. 27	541. 35					

All the last numbers, as you see, are repugnant to harmonic proportions, and seem similar to inexpressibles. For let the number of

[52] Kepler here sets out to explain why harmonies are not to be found in several relations in which they might have been expected to occur, such as the periodic times, the bulks or volumes of the planets, the aphelion and perihelion distances, and finally, the true daily paths at aphelion and perihelion. This leaves the apparent daily motions at aphelion and perihelion as seen from the sun (that is, the angular velocities with respect to the sun) in which at last he locates the celestial harmonies.

[53] Each division of the period by 2 raises the musical interval by an octave. Similarly, multiplication by 2 lowers the interval by an octave. For example, taking one sixteenth of the period of Saturn, namely 672.27 days, raises the interval by four octaves. Then comparing this number of days with the period of Mars gives a ratio of 117:120 approximately. This represents a musical interval within a single octave but it is quite clearly not a consonance.

days for Mars, 687, be measured in units in which it represents 120, which stand for a division of a string. In these units Saturn will be represented by a little more than 117, taking a sixteenth part; Jupiter by less than 95, taking an eighth; the Earth by less than 64; Venus by more than 78, taking double; Mercury by over 61, taking quadruple. Yet these numbers do not make any harmonic proportion with 120; but the neighboring numbers 60, 75, 80, and 96 do. Similarly in units in which Saturn comes to 120, Jupiter comes to about 97, the Earth over 65, Venus more than 80, Mercury less than 63. And in units in which Jupiter comes to 120, the Earth comes to less than 81, Venus less than 100, Mercury less than 78. Also in units in which Venus comes to 120, the Earth comes to less than 98, Mercury to more than 94. Lastly in units in which the Earth comes to 120, Mercury comes to less than 116. But if this free selection of proportions had been valid, they would have been absolutely perfect harmonies, without excesses or deficiencies. God the Creator is therefore not discovered to have intended to introduce harmonic proportions among these sums of times expended added together into periodic times.

Proportion of the planetary globes to each other.

And since it is a very probable conjecture (inasmuch as it depends on geometrical proofs, and on the theory of the causes of the planetary motions set out in the *Commentaries on Mars*) that the bulk of the bodies of the planets are in the proportion of their periodic times,[54] so that the globe of Saturn is about thirty times greater than the globe of the Earth, Jupiter twelve times, Mars less than twice, the Earth greater than one and a half times the globe of Venus, and four times greater than the globe of Mercury, then these proportions of the bodies will not be harmonic either.

Since, however, God has established nothing without geometrical beauty unless it is bound up with some other, prior law of necessity, we readily infer that the periodic times get their durations, and therefore the moving bodies also their bulks, from something which has prior existence in the Archetype. It is to express it that these, as they appear, disproportionate bulks and periods are fitted to this measure. But I have said that the periods are the sum of the times expended, very long, medium, and very slow. The geometrical harmonizations must therefore be found either in these times, or in something prior to them in the mind of the Maker, perhaps. Now the proportions of the expended times are bound up with the proportions of the daily

[54] At this time, Kepler supposed, for archetypal reasons which he does not specify, that the surface area of a planet was proportional to the distance, so that the volume was proportional to the periodic time. In the part of the *Epitome* (KGW 7, pp. 281–282) published in 1620, he abandoned this hypothesis in favor of that of Remus Quietanus, which seemed to be in better accord with the observations; namely, that the volume was in proportion to the distance. It was on the basis of this new hypothesis that he propounded a causal explanation of the harmonic law in the part of the *Epitome* published in 1621 (KGW 7, p. 307).

arcs, because the arcs are in the inverse proportion of the times. Again, we have stated that the proportions of the times expended and the distances of any one planet are the same. As far as individual planets are concerned, therefore, discussion of these three, the arcs, the times expended on equal arcs, and the remoteness of the arcs from the Sun, or the distances, will be one and the same. And because all these are as it happens variable in the case of the planets, there can be no doubt that if they have been assigned any geometrical beauty, by the sure design of the Maker, they acquire it at their extremes, as at their distances in aphelion and perihelion, and not so much at the mean distances in between. For given the proportions of the extreme distances, the design does not need to fit the intermediate proportions to a definite number; for they follow automatically, by the necessity of the planetary motion from one extreme, through all the intermediate points, to the other extreme.

Therefore, the extreme distances are as follows, worked out from the very accurate observations of Tycho Brahe, by the method explained in the *Commentaries on Mars*, by the most persistent exertions of seventeen years.

Distances Compared with Harmonic Intervals

Proportions of pairs			*Distance at:*		*Proportions for individual ones*
Divergent	*Convergent*				
		Of Saturn:	Aphelion	10052.a	More than a minor tone $\frac{10000}{9000}$
			Perihelion	8968.b	Less than a major tone $\frac{10000}{8935}$
$\frac{a}{d}$ $\frac{2}{1}$	$\frac{b}{c}$ $\frac{5}{3}$				
		Of Jupiter:	Aphelion	5451.c	No melodious proportion, but about 11/10, not melodious, or the square root of 6/5, which is harmonic.
			Perihelion	4949.d	
$\frac{c}{f}$ $\frac{4}{1}$	$\frac{d}{e}$ $\frac{3}{1}$				
		Of Mars:	Aphelion	1665.e	Here $\frac{1020}{1388}$ would be harmonic,
			Perihelion	1382.f	$\frac{6}{5}$ and $\frac{1665}{1332}$ would be $\frac{5}{4}$.
$\frac{e}{h}$ $\frac{5}{3}$	$\frac{f}{g}$ $\frac{27}{20}$				
		Of Earth:	Aphelion	1018.g	Here $\frac{1020}{980}$ would be a diesis
			Perihelion	982.h	$\frac{25}{24}$; therefore it does not cover a diesis.
the square root of $\frac{g}{k}$ $\frac{2}{1}$	i.e., $\frac{10000}{7071}$ $\frac{h}{i}$ $\frac{27}{20}$				
		Of Venus:	Aphelion	729.i	Less than a comma and a half:
			Perihelion	719.k	more than a third of a diesis.
$\frac{i}{m}$ $\frac{12}{5}$	$\frac{k}{l}$ $\frac{243}{160}$				
		Of Mercury:	Aphelion	470.l	More than an oversize fifth
			Perihelion	307.m	$\frac{243}{160}$; less than the harmonic $\frac{8}{5}$.

Then there is no single planet of which the extreme distances hint at harmonies, except for Mars and Mercury.

But if you compare the extreme distances of different planets with each other, some light of harmony now begins to shine forth. For the divergent extremes of Saturn and Jupiter make a little more than a diapason; their convergent extremes the mean between major and minor sixths. Similarly the divergent extremes of Jupiter and Mars embrace about a double diapason, and their convergent extremes about a diapason and a diapente. However, the divergent extremes of the Earth and Mars have embraced rather more than a major sixth, and their convergent extremes an oversize diatessaron. In the following couple of the Earth and Venus again there is the same oversize diatessaron between their convergent extremes, but between their divergent extremes we are deserted by harmonic proportion; for it is less than half a diapason (if we may use the phrase), that is, less than the semiduplicate proportion.[55] Lastly, between the divergent extremes of Venus and Mercury the proportion is a little less than the combination of a diapason and a minor third; between their convergent extremes is an oversize diapente, and a little over.

The stages by which the true celestial harmonics were reached. The analogy between a convex orbit and a circular line is the same as between solid figures and harmonics, also the same as between a body and the motion of a body.

Therefore, although one interval departs a little too far from the harmonic proportions, yet this good result was an invitation to proceed further. Now the following was my reasoning. First, these distances, insofar as they are lengths without motion, are not appropriate to be examined for harmonies, because the harmonies are more intimately connected with motion, on account of its swiftness and slowness. Second, in the case of the same distances, insofar as they are diameters of spheres, it is easy to believe that the ratio of the five regular solids should be taken in preference, by analogy. For the ratio of the solid geometrical bodies to the celestial spheres, either enclosed on all sides by celestial matter, as antiquity would have it, or to be enclosed by the accumulation of a great many successive rotations, is also the same as that of the plane figures which are inscribed in a circle (and which are the figures which generate the harmonies) to the celestial circles of the motions, and to the other spaces in which the motions occur. Therefore, if we are seeking harmonies, let us seek for them not in these latter distances, as they are the semidiameters of spheres, but in the former distances, as they are the measures of the motions, that is, rather in the actual motions. Certainly no other distances can be taken as the semidiameters of the spheres, but the average distances; whereas we are dealing with the extreme distances. Therefore, we are not dealing with the distances in respect of their spheres, but in respect of the motions.

For these reasons, then, since I had gone over to comparison of the extreme motions, at first the proportions of the motions remained the same in magnitude as those of the distances were previously, except that they were inverted. Hence some proportions were also found

[55] That is, the proportion $\sqrt{2}$:1.

between the motions, as previously, to be unmelodic, and foreign to the harmonies. However, again I thought that I deserved that result, inasmuch as I was comparing arcs of the eccentric with each other, which are not expressed or counted by a measure of the same size, but are counted in degrees and minutes which are different in size for different planets. Also they do not anywhere show the apparent size which the numerical value of each indicates, except only at the center of each eccentric, which is not supported by any body; and similarly also it is incredible that there should be any sensation or natural instinct in that position in the world which could grasp this apparent size, or rather it is even impossible, if I was comparing the eccentric arcs of different planets, with respect to their apparent sizes at their own centers, which are different in different cases. However, if the different apparent magnitudes were compared, they ought to be apparent at a single position in the world, in such a way that that which has the opportunity of comparing them would be situated at that position of their common appearance. Therefore, I judged that the apparent sizes of these eccentric arcs should either be put out of my mind or represented in a different way. But if I were to put the apparent sizes out of my mind, and turn my attention to the actual daily paths of the planets, I saw that I should have to apply the precept which I stated in Number IX of the previous chapter.[56] Therefore, on multiplying the daily arcs of the eccentrics by the mean distances of the orbits, the following paths resulted.[57]

Thus Saturn completes hardly a seventh of the path of Mercury, and what Aristotle in Book II of his *De Caelo* judged agreeable to reason, that always that which is nearer to the Sun completes a greater distance than that which is further—which cannot be brought about in the ancient astronomy.

		Daily motions. Min. Sec.	*Average distances.*	*Daily paths.*
Of Saturn	at Aphelion	1. 53.	9510.	1075
	at Perihelion	2. 7.		1208
Of Jupiter	at Aphelion	4. 44.	5200.	1477
	at Perihelion	5. 15.		1638
Of Mars	at Aphelion	28. 44.	1524.	2627
	at Perihelion	34. 34.		3161
Of Earth	at Aphelion	58. 6.	1000.	3486
	at Perihelion	60. 13.		3613
Of Venus	at Aphelion	95. 29.	724.	4148
	at Perihelion	96. 50.		4207
Of Mercury	at Aphelion	201. 0.	388.	4680
	at Perihelion	307. 3.		7148

[56] The point of this precept was to have a common measure of the true daily paths, applicable to all planets, and this is obtained by forming the product of the true daily arcs (measured in minutes and seconds of angle subtended at the center of the eccentric) by the mean distance of the planet from the sun.

[57] The true daily path in the third column is obtained by multiplying the true daily motion in seconds of arc by the average distance in the second column and then dividing by 1000 for convenience. The resulting numbers represent the true daily paths for all the planets in a common measure.

Thus Saturn completes scarcely a seventh part of the path of Mercury; and the result is what Aristotle in Book II of his *De Caelo* (*On the Heaven*)[58] judged to be in agreement with reason, that the planet which is nearer to the Sun always completes a greater distance than the one which is further away, which is impossible to attain in the ancient astronomy.

Therefore, as far as the daily paths of individuals are concerned, the proportions which they comprise ought to be the same in magnitude as those which were previously in the distances, but inverted in kind, because the eccentric arcs, as has been stated, have the inverse proportion of their own distances from the Sun.

However, if we consider the extreme paths of the pairs, either divergent or convergent, there is much less appearance of anything harmonic than previously when we had considered the actual arcs.

And indeed if we should ponder the matter more carefully, it will be apparent that it is not very likely that the most wise Creator should have taken thought most of all for harmonies between the actual planetary paths. For if the proportions of the paths are harmonic, all the other features of the planets will be constrained, and linked to the paths, so that there will be no room for taking thought for harmonies elsewhere. But who will benefit from harmonies between the paths, or who will perceive these harmonies? There are two things which reveal to us harmonies in natural occurrences, either light or sound. The former is received through the eyes, or hidden senses analogous to eyes, the latter through the ears; and the mind seizing on these emanations distinguishes either by instinct (on which plenty has been said in Book IV) or by astronomical or harmonic reasoning between melodic and unmelodic. In fact, no sounds exist in the heaven, and the motion is not so turbulent that a whistling is produced by friction with the heavenly air. There remains light. If it is to teach us anything about the paths of the planets, it will teach us that either the eyes, or some sensory organ analogous to them, are located in a certain position; and for the light to inform us immediately of its own accord, it seems that the sensory organ must be there in its presence. Therefore, there will be an organ of sensation all over the world, that is to say in such a way that one and the same is present to the motions of all the planets. For that way which was traversed by dint of observations, by way of long drawn out wanderings in geometry and arithmetic, of the proportions of the spheres and the rest which had to be learnt beforehand, to reach these actual paths, is too long for some natural instinct, to influence which it seems to be fitting that the harmonies were introduced.

Therefore, assembling all these points into a single review I have rightly concluded that we should dismiss the true paths of the planets

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[58] Aristotle, *De caelo*, 291 a 29–291 b 10.

through the aethereal air, and turn our eyes to the apparent daily arcs, all indeed apparent to one definite and prominent position in the world, that is to say to the actual solar body, the source of the motion of all the planets. Also we should look not how high any particular planet is from the Sun, nor what space it traverses in a single day—for that is rational and astronomical, not instinctive—but how large an angle the daily motion of each planet subtends at the actual body of the Sun, or how large an arc on one common circle drawn about the Sun, such as the ecliptic, it seems to complete on any particular day. Thus this appearance, brought by the agency of light to the body of the Sun, can along with the light itself flow straight to living creatures, who share in this instinct, just as in the fourth book we have stated that the pattern of the heaven flows to a foetus by the agency of the rays.[59]

Therefore, the Tychonic astronomy teaches us (abstracting from the proper motion of the planets the parallaxes of the annual orbit, which impart to them the semblance of stations and retrogressions) that the daily motions of the planets in their own orbits (as they appear, so to speak, to those watching on the Sun) are as follows:

Harmonies of pairs.				*Apparent daily paths.*		*Individuals' own harmonies.*	
Div.	*Conv.*			*Min. Sec.*		*Min. Sec.*	
		Saturn	at Aphelion	1.46.a	Between	1.48.	is $\frac{4}{5}$ a major third.
			at Perihelion	2.15.b	and	2.15.	
a/d $\frac{1}{3}$	b/c $\frac{1}{2}$						
		Jupiter	at Aphelion	4.30.c	Between	4.35.	is $\frac{5}{6}$ a minor third.
			at Perihelion	5.30.d	and	5.30	
c/f $\frac{1}{8}$	d/e $\frac{5}{24}$						
		Mercury	at Aphelion	26.14.e	Between	25.21	is $\frac{2}{3}$ a diapente.
			at Perihelion	38. 1.f	and	38.1	
e/h $\frac{5}{12}$	f/g $\frac{2}{3}$						
		Earth	at Aphelion	57. 3.g	Between	57.28	is $\frac{15}{16}$ a semitone.
			at Perihelion	61.18.h	and	61.18	
g/k $\frac{3}{5}$	h/i $\frac{5}{8}$						
		Venus	at Aphelion	94.50.i	Between	94.50	is $\frac{24}{25}$ a diesis.
			at Perihelion	97.37.k	and	98.47	
i/m $\frac{1}{4}$	k/l $\frac{3}{5}$						
		Mars	at Aphelion	147. 0.l	Between	164. 0	is $\frac{5}{12}$ a diapason and minor third
			at Perihelion	384. 0.m	and	394. 0	

[59] Here Kepler seems to suggest that the reception of the celestial harmonies by living creatures is instinctive, like that of the aspects, the harmonies being conveyed in some way along with the light from the sun. Of the various possible locations of the celestial harmonies, Kepler indicates that the one which places them in the apparent motions (as seen from the sun) is the one which would need the least amount of calculation and discursive reasoning for their recognition. In other words, this is the most suitable location for an instinctive recognition.

Notice that the great eccentricity of Mercury makes the proportion of the motions differ considerably from the square of the proportion of the separations.[60] For if you make the proportion of the motion at aphelion to the mean motion, 245 minutes 32 seconds, that is the square of the proportion of the mean separation, taken as 100, to the separation at aphelion, 121, then the resulting motion at aphelion is 167; and if you make the proportion of the motion at perihelion to the same mean motion, that is the square of the proportion of 100 to the distance at perihelion, 79, the motion at perihelion will be made 393. In both cases it is greater than I have supposed here, naturally because the mean motion at the mean anomaly being viewed very obliquely does not appear as great, that is to say not 245 minutes 32 seconds but smaller by about 5 minutes. Therefore, the motions at aphelion and at perihelion will also be found to be smaller. However, it will be less so for the motion at aphelion, and more so for the motion at perihelion, on account of the theorem in Euclid's *Optics*, in accordance with my warning in the preceding chapter, under Number VI.

Therefore, I could assume mentally that between these apparent extreme motions of individual planets there would be harmonies, and their distances would be melodic, and that indeed from the proportions of their daily eccentric arcs, set out above, since I there saw that square roots of harmonic proportions reigned everywhere, whereas I knew that the proportion of the apparent motions was the square of that of the eccentric motions. But we may verify what is stated by actual observation, indeed without reasoning, as you see in the next table. For the proportions of the apparent motions of individual planets come very close to harmonies. Thus Saturn and Jupiter embrace a very little more than thirds, major and minor: there is an excess in the former case of 53:54, in the latter of 54:55 or less, that is to say about one and a half commas; the Earth embraces a very little more, that is to say 137:138 more, scarcely half a comma, than a semitone; Mars somewhat (that is to say 29:30, which is close to 35:35 or 35:36) less than a diapente; Mercury occupies, over the diapason, nearer a minor third than a tone, that is to say it has less by about 38:39, which is about two commas, in other words about 34:35 or 35:36. Venus alone occupies something smaller than any of the melodic intervals, and is itself just a diesis; for its proportion is between two and three commas, and exceeds two thirds of a diesis, being about 34:35, almost 35:36, a Diesis diminished by a comma.

The Moon also enters into consideration here.[61] For it is found that its hourly motion at apogee in quadrature, that is to say when

[60] Kepler has shown that, for small eccentricities, the apparent angular velocity, as seen from the sun, is inversely proportional to the square of the distance from the sun. Cf. Chapter III, number six and note 41.

[61] The proportion of the moon's apparent motion is taken to be that as seen from the earth.

it is slowest of all, is 26 minutes 26 seconds. At perigee at the syzygies, that is to say when it is fastest of all, it is 35 minutes 12 seconds. By this ratio a diatessaron is formed with great exactness. For a third part of 26′26″ is 8′49″—four times which is 35′15″. And notice that the harmony of diatessaron is found nowhere else among the apparent motions. Notice also the analogy of the Fourth in the harmonies with quadrature in the phases.[62] These, then, are found in the motions of individual planets.

What the harmonies are between the approaching and receding motions of the pairs.

But among the extreme motions of the pairs of planets compared with each other, the clearest light is thrown at once as soon as we look at the heavenly harmonies, whether you compare the receding extreme motions with each other, or the approaching. For between the approaching motions of Saturn and Jupiter the proportion is exactly double, or a diapason; between their receding motions, it is a very little more than triple, or a diapason with a diapente. For of 5 minutes 30 seconds, a third part is 1 minute 50 seconds, whereas Saturn has instead of that 1 minute 46 seconds. Therefore, the planetary proportion has one diesis over, or something a little less, that is 26:27 or 27:28; and when Saturn is approaching to within less than a single second from aphelion, the excess will be 34:35, the size of the proportion between the extreme motions of Venus. Between the diverging and converging motions of Jupiter and Mars reign the triple diapason, and the third a double diapason above, though not perfectly. For an eighth part of 38 minutes 1 second is 4 minutes 45 seconds, whereas Jupiter has 4 minutes 30 seconds. Between those numbers there is still a difference of 18:19, which is the mean between 15:16 and 24:25, a semitone and a diesis, that is to say very nearly a perfect limma, 128:135. Similarly, a fifth part of 26 minutes 14 seconds is 5 minutes 15 seconds, whereas Jupiter has 5 minutes 30 seconds. Therefore, the deficiency from the fivefold proportion here is about 21:22, the amount of the excess in the other proportion previously, that is about a diesis, 24:25. The harmony 5:24 which takes in a minor instead of a major third over the second octave comes rather near. For of 5′30″ a fifth part is 1′6″, and taking twenty four times that produces 26′24″, with which 26′14″ makes no more than half a comma.[63] Mars has been allotted a very small proportion with the Earth, very exactly the sesquialterate, or a diapente; for a third part of 57 minutes 3 seconds is 19 minutes 1 second, and double that is 38 minutes 2 seconds, the very number which Mars has, that is 38 minutes 1 second. As their greater proportion they have been allotted a diapason with

See Book III, Chapter IV, page 182

[62] The absence of a primary consonance like the diatessaron from the celestial harmonies would, of course, have been a serious problem for Kepler, so he was pleased to find this harmony in the apparent motion of the moon and a reason for its location here in the analogy with the quadratures in the phases.

[63] For purposes of calculation, Kepler later represents half a comma by the proportion 157:158.

a minor third, 5:12, a little less nearly perfect. For a twelfth part of 61 minutes 18 seconds is 5 minutes $6\frac{1}{2}$ seconds, and taking five times that gives 25 minutes 33 seconds, whereas instead of that Mars has 26 minutes 14 seconds. The deficiency is therefore about a narrow diesis, that is 35:36. However, the Earth and Venus have been allotted harmonies in common, the greatest 3:5 and the least 5:8, which are sixths, major and minor, again not quite perfect. For a fifth part of 97 minutes 37 seconds is 19 minutes 31 seconds, and three times that comes to 58 minutes 34 seconds which is more than the motion of the Earth at aphelion by 34:35, which is almost 35:36, the amount by which the planetary proportion exceeds the harmonic. Similarly, an eighth part of 94 minutes 50 seconds is 11 minutes 51 seconds +, and five times that is 59 minutes 16 seconds which is as nearly as possible equal to the mean motion of the Earth. Hence in this case the planetary proportion is less than the harmonic by 29:30, or 30:31, which again is nearly 35:36, a narrow diesis; and to that extent this smallest of their proportions approaches the harmony of diapente. For a third part of 94 minutes 50 seconds is 31 minutes 37 seconds, and twice that is 63 minutes 14 seconds, from which the motion of the Earth at perihelion, 61 minutes 18 seconds, is deficient by the tiny amount of 31:32, so that the planetary proportion occupies exactly the mean between the neighboring harmonic proportions. Lastly, the proportions allotted to Venus and Mercury are as the greatest a double diapason, and as the least a hard sixth, though these are not absolutely perfect. For a fourth part of 384 is 96 minutes 0 seconds, whereas Venus has 94 minutes 50 seconds. Therefore, it approaches the fourfold within about one comma. Similarly, a fifth part of 164 minutes is 32 minutes 48 seconds, and taking three times that makes 98 minutes 24 seconds, whereas Venus has 97 minutes 37 seconds. Therefore, the planetary proportion is in excess by about two thirds of a comma, that is 126:127.

These, then, are the harmonies with each other allocated to the planets; and there is none of the direct comparisons (that is to say between convergent and divergent extreme motions) which does not come very close to some harmony, so that if strings were tuned in that way, the ears would not easily be able to detect the imperfection, except for the excess of the single one between Jupiter and Mars.[61]

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[61] Kepler here takes the maximum imperfection "which the ears would not easily be able to detect" as the diesis 24:25. Only in the case of the divergent motions of Jupiter and Mars is the imperfection greater than a diesis. In this case it is a limma 128:135, which is the mean between a diesis and a semitone. Although the imperfections are small, they are still not as small as Kepler could have wished. For an imperfection as large as a diesis would not be tolerated in musical performance, where the largest acceptable imperfection is the comma 80:81, which is less than a third of the diesis.

What the harmonies are between the motions of the pairs on the same side.

Now it follows that if we compare the motions on the same side[65] we shall not be likely to stray far from the harmonies in that case either. For on multiplying the 4:5 times 53:54 of Saturn by the intermediate proportion 1:2 the combined product is 2:5 times 53:54, which is the proportion between the motions at aphelion of Saturn and Jupiter.[66] Multiply by the 5:6 times 54:55 of Jupiter: the product is 5:12 times 54:55 which is the proportion between the motions at perihelion of Saturn and Jupiter. Similarly multiply the 5:6 times 54:55 of Jupiter by the following intermediate proportion, 5:24 divided by 157:158; the result is 1:6 divided by 35:36, the proportion between the motions at aphelion.[67] Multiply the same, 5:24 divided by 157:158, by the 2:3 divided by 29:30 of Mars; the result is 5:36 divided by 24:25, about, that is 125:864 or nearly 1:7, the proportion between the motions at perihelion: in fact this alone so far is unmelodic.[68] Multiply the third of the intermediate proportions, 2:3,[69] by the 2:3 divided by 29:30 of Mars: it comes out as 4:9 divided by 29:30, that is 40:87, another unmelodic interval between the motions at aphelion. If instead of the proportion for Mars you multiply by the Earth's 15:16 times 137:138 you will obtain 5:8 times 137:138, the proportion between them at perihelion.[70] And if you multiply the fourth of the intermediate proportions, 5:8 divided by 30:31, or 2:3 times 31:32, by the Earth's 15:16 times 137:138, you will find the product is very nearly 3:5, the proportion between the motions at aphelion of the Earth and Venus.

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65 That is, comparing the motions of the two planets at aphelion or at perihelion.

66 Kepler has already shown that the proportion of the aphelion and perihelion motions of Saturn exceeds the major third 4:5 by 53:54, or about one and a half commas. Combining these intervals with the almost exact octave 1:2 between the perihelion motion of Saturn and the aphelion motion of Jupiter gives an interval 53:54 in excess of 2:5 for the proportion of the motions of Saturn and Jupiter at aphelion. The interval between these aphelion motions thus exceeds the consonance of an octave and a major third by about one and a half commas. In this case, therefore, the imperfection is well within the limit of a diesis that Kepler is willing to accept. The further calculations described by Kepler proceed in the same way. By adopting this method of calculation, instead of simply calculating the proportion of the motions from the table given earlier, Kepler is able to estimate the imperfections in the new intervals by using those already found in the intervals of the motions of each planet and the intervals of the divergent and convergent motions. Intervals are combined, it should be noted, by multiplying the proportions.

67 That is, the proportion of the aphelion motions of Jupiter and Mars. This is equivalent to a musical interval of about two commas less than two octaves and a fifth.

68 The proportion of the perihelion motion of Jupiter and the aphelion motion of Mars is 5:24 divided by the imperfection 157:158. Combining this with the proportion of the aphelion and perihelion motions of Mars, 2:3 divided by the imperfection 29:30, gives the proportion of the perihelion motions of Jupiter and Mars which, as Kepler notes, corresponds to the dissonant interval 1:7.

69 That is, the proportion of the perihelion motion of Mars and the aphelion motion of the Earth.

70 Thus the proportion of the perihelion motions of Mars and the Earth corresponds to a consonant interval, with an imperfection of about half a comma.

For a fifth part of 94 minutes 50 seconds is 18 minutes 58 seconds, and three times that is 56 minutes 54 seconds, whereas the Earth has 57 minutes 3 seconds.[71] If you multiply the same proportion by the 34:35 of Venus,[72] you obtain a product of 5:8, the proportion between the motions at perihelion. For an eighth part of 97 minutes 37 seconds is 12 minutes 12 seconds +, and taking five times that gives a return of 61 minutes 1 second, whereas the Earth has 61 minutes 18 seconds.

Lastly, if you multiply the last of the intermediate proportions, 3:5 times 126:127, by Venus' 34:35, the combined product will be 24:25 times 3:5, and the result is a dissonant interval, made of the two combined, between the motions at aphelion. Nevertheless, if you multiply by Mercury's proportion, 5:12 divided by 38:39, now it will fall short of 1:4, or the double diapason, by as nearly as possible a complete diesis, for the proportion between the motions at perihelion.

Therefore, perfect harmonies are found between the convergent extreme motions of Saturn and Jupiter, a diapason; between the converging extremes of Jupiter and Mars, a double diapason together with nearly a soft third; between the converging extremes of Mars and the Earth, a diapente, and between their motions at perihelion a soft sixth; between the motions of the Earth and Venus at aphelion, a hard sixth, and at perihelion a soft sixth; between the converging extremes of Venus and Mercury a hard sixth, and between their divergent extremes or even between their motions at perihelion, a double diapason.[73] Hence without detriment to the astronomy developed most subtly of all from the observations of Brahe, it seems that the residual very tiny discrepancy can be absorbed, especially in the motions of Venus and Mercury.

However, you will notice that where there is not a perfect major harmony, as between Jupiter and Mars, there alone I have detected a very nearly perfect intermediate placing of the solid figure, since the separation of Jupiter at perihelion is very nearly three times that of Mars at aphelion, so that this pair aspires in its distances to the perfect harmony which it has not got in its motions.[74] You will no-

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[71] The imperfection is about a quarter of a comma.

[72] Kepler has already shown that the proportion of the aphelion and perihelion motions of Venus corresponds to a diesis diminished by a comma; that is, the interval 34:35.

[73] With the exception of the proportion of the perihelion motions of Venus and Mercury, where the imperfection is a diesis, the proportions mentioned here by Kepler represent harmonies that are either perfect or well within a comma of absolute perfection; in other words, they represent intervals acceptable as perfect in musical performance.

[74] Kepler explains discrepancies in the interpolation of the regular polyhedra between the planetary spheres as a consequence of the requirements of the celestial harmony. Conversely, where the harmonies fail, there must be a reason, and this he located in the interpolation of the solids. Nothing is the result of chance or

tice further that the greater planetary proportion of Saturn and Jupiter exceeds the harmonic proportion, that is to say the threefold, by almost the same amount as is Venus' own proportion; and the deficiency in the common greater proportion of Mars and the Earth is also almost the same as that in the two common proportions of the extremes of the Earth and Venus, convergent and divergent. You will notice thirdly that among the superior planets there are almost fixed harmonies between the convergent motions, but among the inferior planets between motions in the same direction.[75] And notice fourthly that between the motions at aphelion of Saturn and the Earth there are very nearly five diapasons; for a thirty second part of 57 minutes 3 seconds is 1 minute 47 seconds, whereas the motion of Saturn at aphelion amounts to 1 minute 46 seconds.

Further, there is a great distinction between the harmonies which have been set out between individual planets, and between planets combined. For the former cannot indeed exist at the same moment of time, whereas the latter can absolutely; because the same planet when it is situated at its aphelion cannot at the same time also be at its perihelion which is opposite, but of two planets one can be at its aphelion and the other at its perihelion at the same moment of time.[76] Then the proportion of simple melody or monody, which we call choral music and which was the only kind known to the ancients, to the melody of several voices, called figured and the invention of recent centuries, is the same as the proportion of the harmonies which are indicated by individual planets to the harmonies which they indicate in combination. Further, then, in Chapters V and VI the individual planets will now be compared with the choral music of the ancients, and its properties will be demonstrated in the motions of the planets; but in the chapters which follow it will be demonstrated that the planets in combination match modern figured music.

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accident. So, where harmony is lacking in certain motions, such as those of Jupiter and Mars to which Kepler refers, the planets must have been arranged in accordance with some other principle; in this case the harmony of distances arising from the interpolation of the polyhedra. The geometrical archetype is a composite one, consisting of the regular polyhedra and the harmonic proportions.

[75] That is, the motions of the two planets at aphelion or at perihelion.

[76] The notes of the harmonic intervals represented by the single planets can only be sounded in succession, as in a melody consisting of a single line. The notes of the harmonies represented by pairs of planets, however, can be sounded simultaneously, as in the polyphonic music that Kepler believed to be a recent invention.

Chapter V.

That the Positions in the System, or the Notes[77] of the Musical Scale, and the Kinds of Melody, Hard and Soft, Have Been Expressed in the Apparent (to observers on the sun, so to speak) Planetary Motions.

Therefore, that between these twelve terms or motions of the six planets which revolve round the Sun there exist upwards, downwards, and in every direction proportions which are harmonic, or very close to such within an imperceptible fraction of the smallest melodic interval, has been proved so far by numbers which have been sought in the former case from astronomy and in the latter from harmony. However, just as in the third Book we first extracted the individual harmonic proportions separately in the first Chapter, and only after that in the second Chapter we assembled all that there were of them into one common system or musical scale, or rather we divided one diapason of them, which embraces the remaining ones in its dominion, through those remaining ones into steps or positions, so that by this procedure we produced a scale; so also now, when we have found the harmonies which God Himself embodied in the world, the next thing is for us to see whether the individual harmonies stand separately, so that they have no affinity with the rest, or whether in fact they all agree with each other? However, it is easy without further inquiry to conclude that these harmonies are fitted together with the utmost skill so that they support each other mutually as if within a single structure, and no single one clashes with another, inasmuch as we see that in such a many-sided comparison of their terms harmonies never fail to occur. For if all were not fitted to all to form a single scale, it could easily have come about (as has happened here and there, when necessity is so pressing) that several dissonances occurred. Thus if anyone established a major sixth between the first and second term, and between the second and third a third, also major, without regard to the previous interval, in that case he would be admitting between the first and the third a dissonance and an unmelodic interval, 12:25.[78]

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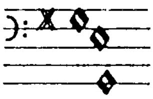
[77] The word "clavis" (literally "key") here means "note."

[78] Kepler's purpose here is simply to show that if harmonies are combined without regard to the way in which they fit together to form a system, dissonances would be generated accidentally.

Come now, let us see whether what we have already inferred by reasoning is in actual fact found to be so. However, let us preface this



with some words of caution, to avoid our being obstructed while the inquiry is in progress. First, we should for the present overlook those excesses, or deficiencies, which are less than a semitone; for we shall see later what causes them. Next by repeated doubling, or on the contrary halving, of the motions, we shall bring them all within a system of a single octave, because of the identity of sound of every diapason.[79]

Therefore, the numbers by which all the positions or notes of the system of the octave are expressed are set out in the table in Chapter VIII of Book III, on page 197: that is to say, those numbers should be understood to refer to the length of the pairs of strings. In consequence, therefore, the speeds of the motions will be to each other inversely in those proportions.[80]

Now let the motions be compared, in parts obtained by continuous division by two.[81]

Then of the motion at		*min.*	*sec.*	
Perihelion of Mercury the seventh halving, or 128th	is	3.	0.	
Aphelion of Mercury the sixth halving, or 64th	is	2.	34.	–
Perihelion of Venus the fifth halving, or 32nd	is	3.	3.	+
Aphelion of Venus the fifth halving, or 32nd	is	2.	58.	–
Perihelion of Earth the fifth halving, or 32nd	is	1.	55.	–
Aphelion of Earth the fifth halving, or 32nd	is	1.	47.	–
Perihelion of Mars the fourth halving, or 16th	is	2.	23.	–
Aphelion of Mars the third halving, or 8th	is	3.	17.	–
Perihelion of Jupiter half	is	2.	45.	
Aphelion of Jupiter half	is	2.	15.	
and the motion at				
Perihelion of Saturn	is	2.	15.	
Aphelion of Saturn	is	1.	46.	

[79] Intervals of one or more octaves are identical by opposition. See Book III, Chapter 1, proposition 1.

[80] The proportion of the motions is the inverse of that of the lengths of the strings because the greater motion corresponds to the higher pitch and consequently to the shorter string.

[81] In the table that follows, the motions of Saturn are left unchanged. The motions of Jupiter are divided by 2, which has the effect of lowering the corresponding notes by an octave. The aphelion motion of Mars is divided by 8, thus lowering the note by three octaves, while the perihelion motion is divided by 16, equivalent to a lowering by 4 octaves. The motions of the Earth and Venus are divided by 32, equivalent to a lowering by five octaves. The aphelion motion of Mercury is divided by 64 and the perihelion motion by 128, equivalent respectively to a lowering by six and seven octaves. This brings the notes of all the planets within a single octave. The plus and minus signs indicate insignificant differences arising from approximation to the nearest second in the division.

Now let the motion of Saturn, the slowest planet, at aphelion, that is the slowest motion, represent the lowest position in the system, *G*, numerically 1 minute 46 seconds. Then the motion at aphelion of the Earth will also represent the same, but five diapasons higher, because it is numerically 1 minute 47 seconds; and who would venture to argue about one second in the motion of Saturn at aphelion? However, let it stand: the difference will not be greater than 106:107, which is less than a comma. Of this 1 minute 47 seconds, if you add a fourth part, 27 seconds, the total will be 2 minutes 14 seconds, whereas the motion of Saturn at perihelion comes to 2 minutes 15 seconds.[82] Similarly, for the motion of Jupiter at aphelion, but one diapason higher. Therefore, these two motions represent the note 𝔥, or are very slightly higher. Take a third of 1 minute 47 seconds, 36 seconds—and add it to the whole: you will generate 2 minutes 23 seconds—standing for the note *c*; and look, it is the motion of Mars at perihelion, of the same magnitude, but four diapasons higher.[83] To the same 1 minute 47 seconds add also a half, 54 seconds—the result will be 2 minutes 41 seconds –, standing for the note *d*; and look, here to hand is the motion of Jupiter at perihelion, but one diapason higher: for it takes a number which is very close, that is 2 minutes 45 seconds.[84] If you add two thirds, that is 1 minute 11 seconds +, the sum is 2 minutes 58 seconds +. And look, the motion of Venus at aphelion is 2 minutes 58 seconds –. Therefore, this represents the position or note *e*, but five diapasons higher; and the motion of Mercury at perihelion does not greatly exceed it, having 3 minutes 0 seconds, but seven intervals of a diapason higher.[85] Lastly, divide twice 1 minute 47 seconds, that is 3 minutes 34 seconds, by nine, and subtract one ninth, 24 seconds, from the whole. The remainder is 3 minutes 10 seconds +, standing for the note *f*,[86] which is nearly represented by the motion of Mars

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[82] Adding the fourth part is equivalent to increasing in the ratio 4:5, the proportion of the major third, so that, if the lower note is taken as *G*, the higher note will be 𝔥. The motions of Saturn at perihelion and Jupiter at aphelion represent a note higher than 𝔥 by the interval 134:135, which is less than a comma.

[83] Adding the third part is equivalent to increasing in the ratio 3:4, the proportion of the fourth, and since the lower note is taken as *G*, the upper note is *c*. When Kepler takes a third of 1 minute 47 seconds, he describes the result as 36 seconds –. It is in fact $35\frac{2}{3}$ seconds, so that his minus sign denotes an insignificant difference arising from approximation to the nearest second. In what follows, the signs should be interpreted in the same way.

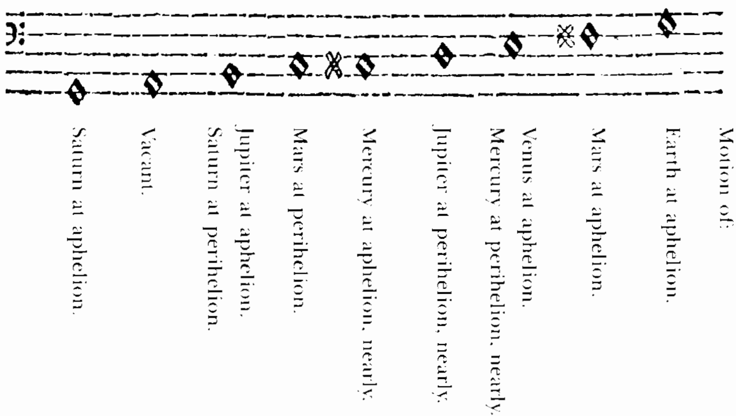
[84] The perihelion motion of Jupiter in fact represents a note about 2 commas higher than *d*.

[85] The perihelion motion of Mercury represents a note less than a comma higher than *e*.

[86] The note *f* is a semitone above *e*, which is a major sixth above *G*. The interval from *G* to *f* is therefore represented by the product of the proportions 3:5 and 15:16, that is 9:16. By his subtraction, Kepler obtains 8/9ths of 3 minutes 34 seconds, which is 16/9ths of 1 minute 47 seconds. The proportion of the motions 1 minute 47 seconds and 3 minutes 10 seconds is therefore 9:16, so that, if the slower motion is associated with the note *G*, the quicker motion corresponds to *f*.

at aphelion, 3 minutes 17 seconds, but three diapasons higher; but the actual number is a little greater than it should be, coming close to the note *f*.[87] For taking a sixteenth of 3 minutes 34 seconds, that is $13\frac{1}{2}$ seconds, from 3 minutes 34 seconds leaves 3 minutes $20\frac{1}{2}$ seconds, to which 3 minutes 17 seconds is very close. And indeed in music also *f*ϱ is often used in place of *f*, as may be seen everywhere.[88]

Therefore, all the notes in hard music within a single octave (except for the note *A*, which was not represented by the harmonic divisions in Book III, Chapter II, either), are represented by all the extreme motions of the planets, except for the motions at perihelion of Venus and the Earth, and the motion at aphelion of Mercury, for which the number is 2 minutes 34 seconds which is close to the note *c*ϱ. For subtract from *d*, 2 minutes 41 seconds, a sixteenth, 10 seconds + : the remainder is 2 minutes 30 seconds, for the note *c*ϱ. Thus only the motions at perihelion of Venus and the Earth are outcasts from this scale, as you see in the figure.[89]



On the other hand, if the motion of Saturn at perihelion, 2 minutes 15 seconds, is made the start of the scale, and it is directed that it should represent the note *G*; then the note *A* fits 2 minutes 32 seconds –, which is very close to the motion at aphelion of Mercury.[90] The note *b* fits 2 minutes 42 seconds, which is very nearly the motion

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[87] The aphelion motion of Mars represents a note nearly 3 commas above *f* but just over a comma below *f*ϱ.

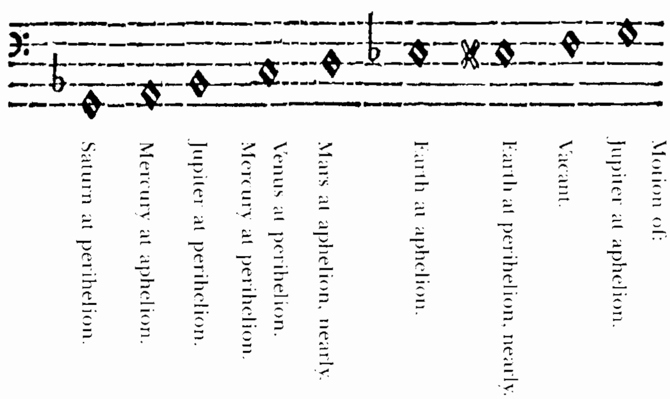
[88] In Kepler's hard scale note VII is taken to be *f*, though in musical practice *f*ϱ was frequently used instead, as in the modern major scale.

[89] Apart from the two outcasts, the motions of the planets fit the notes to within one comma, with the exception of the perihelion motion of Jupiter where the discrepancy is two commas. Although Kepler has marked some of the motions as only an approximate fit, there is no justification for this distinction, since a difference of a comma is acceptable to the ear.

[90] The aphelion motion of Mercury corresponds to a note just over a comma higher than *A*.

at perihelion of Jupiter, by the equivalence of octaves.[91] The note *c* fits 3 minutes 0 seconds, the motion at perihelion of Mercury and Venus, very nearly.[92] The note *d* fits 3 minutes 23 seconds—and the motion of Mars at aphelion is not much lower, that is 3 minutes 18 seconds. Thus this number is smaller than its note by almost the same amount as that by which previously it was in a similar way greater than its note.[93] The note *d*ϱ fits 3 minutes 36 seconds, which the motion of the Earth at aphelion almost meets;[94] the note *e* fits 3 minutes 50 seconds and the motion of the Earth at perihelion is 3 minutes 49 seconds.[95] However, the motion of Jupiter at aphelion again occupies *g*.

On this basis all the notes within one octave of soft music, except for *f*, are expressed by most of the motions of the planets at aphelion and perihelion, especially those which had been left out previously, as you see in the figure.



Now previously *f*ϱ was represented, *A* was left out; now *A* is represented, *f*ϱ is left out, for the harmonic divisions in Chapter II also left out the note *f*.

Therefore, there has been expressed in the heaven in a twofold way, and in two, so to speak, kinds of melody, the musical scale, or

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[91] The perihelion motion of Jupiter corresponds to a note higher than *b* by about one and a half commas.

[92] In the case of the perihelion motion of Mercury, the correspondence is exact, but in the case of Venus, the perihelion motion corresponds to a note nearly one and a half commas above *c*.

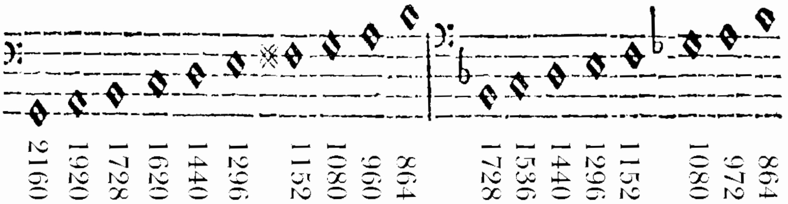
[93] The aphelion motion of Mars corresponds to a note 2 commas lower than *d*. In the case of the hard scale, the aphelion motion of Mars corresponded to a note nearly 3 commas above *f*. Cf. note 87.

[94] Dividing the Earth's aphelion motion, 57 minutes 3 seconds, by 16, equivalent to a lowering by 4 octaves, gives 3 minutes 34 seconds. The note corresponding to the Earth's aphelion motion is therefore less than a comma lower than *d*ϱ.

[95] Kepler has made a numerical slip, for the note *e* fits 3 minutes 45 seconds while the perihelion motion of the Earth is nearly 3 minutes 50 seconds. Instead of the almost perfect fit indicated by Kepler, the Earth's perihelion motion corresponds to a note nearly 2 commas below *e*.

system of one octave, with all the positions by means of which natural melody is conveyed in music. The sole difference is in the fact that in our harmonic divisions indeed both ways jointly start from one and the same term, whereas in the latter case in the motions of the planets what was previously ♮ now in the soft kind becomes *G*.[96]

In the motions of the heavens like this:



Through the harmonic divisions like this:

For just as in Music the proportion is 2160:1800, or 6:5, so in the former system, which is expressed by the heaven, the ratio is 1728:1440, that is also as 6:5, and similarly for several other cases:[97]

	2160	:1800	:1620	:1440	:1350	:1080
as	1728	:1440	:1296	:1152	:1080	:864

You will now therefore wonder no more at the establishment of the most excellent order of the sounds or steps in the musical system or scale by men, since you see that all they are doing in this respect is aping God the Creator, and as it were acting out a particular scenario for the ordering of the heavenly motions.

[96] In the hard scale, taking the aphelion motion of Saturn as *G*, the perihelion motion of Saturn becomes ♮. In the soft scale, the perihelion motion of Saturn is taken as *G*. We may say that the ♮ of the hard scale is transposed to become the higher *G* of the soft scale. The clef at the beginning of the soft scale indicates the transposed scale. If this is ignored, we see the position of the notes in relation to the hard scale.

[97] The relationship also holds for the two cases omitted from this list; that is 2160:1920 = 1728:1536 and 2160:1215 = 1728:972.

In fact there also remains another means by which we may grasp the double musical scale in the heaven, in which the system is indeed the same, but the tuning is conceived in twin ways, one according to the motion of Venus at aphelion, the other according to the motion at perihelion. For the variation in the motions of this planet is of very small extent, inasmuch as it is contained within the size of a diesis, the smallest melodic interval. And the tuning at aphelion indeed, as above, has the motions at aphelion of Saturn, the Earth and Venus, and almost that of Jupiter, at *G*, *e*, and ♮ but the motion at perihelion of Mars, and almost that of Saturn, and as appears at first sight also that of Mercury, at *c*, *e*, and ♮;[98] whereas on the other hand the tuning at perihelion gives a position also to the motions at aphelion of Mars and Mercury and almost that of Jupiter, but to the motions at perihelion of Jupiter, Venus, and almost that of Saturn, but also to a certain extent to that of Earth, and undoubtedly that of Mercury also. For suppose that now it is the motion of Venus not at aphelion but at perihelion, 3 minutes 3 seconds, which occupies the position *e*. The motion at perihelion of Mercury also approaches very close to it, over a double diapason, by the end of Chapter IV. However, if of this motion of Venus at perihelion, 3 minutes 3 seconds, one tenth, 18 seconds, is subtracted, the remainder is 2 minutes 45 seconds, the motion at perihelion of Jupiter, which holds the position *d*; and the addition of a fifteenth, 12 seconds, gives a sum of 3 minutes 15 seconds which is nearly the motion at aphelion of Mars, which holds the position *f*. And similarly in the case of ♮, the result of almost the same tuning is also the motions of Saturn at perihelion and of Jupiter at aphelion.[99] But if an eighth part, 23 seconds, is taken five times, it yields 1 minute 55 seconds, which is the motion of the Earth at perihelion.[100] Although this does not square with the same scale as those which have been mentioned before, inasmuch as it does not include in order the interval 5:8 below *e*, nor 24:25 above *G*, yet if now the motion at perihelion of Venus, and similarly also the motion at aphelion[101] of Mercury, take, out of order, the position *dϱ* instead of *e*, then this motion of the Earth at perihelion will take the position *G*, and the motion of Mercury at aphelion will also agree. For a third

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[98] The correspondences are as follows:

G	♮	*c*	*e*
aphelion Saturn	aphelion Jupiter	perihelion Mars	aphelion Venus
aphelion Earth	perihelion Saturn		perihelion Mercury

[99] Taking the perihelion motion of Venus, 3 minutes 3 seconds, to represent *e*, the note ♮ is represented by 2 minutes 17 seconds. The aphelion motion of Jupiter and the perihelion motion of Saturn represent a note just over a comma lower.

[100] By this calculation the Earth's aphelion motion is represented by a note a minor sixth below *e* or a diesis above *G*. For the sum of these intervals is the major sixth between *G* and *e*. But, as Kepler goes on to point out, such a note does not belong to the scales he has so far described.

[101] Kepler intends the motion of Mercury at perihelion.

of 3 minutes 3 seconds, 1 minute 1 second, taken five times, becomes 5 minutes 5 seconds, half of which, 2 minutes 32 seconds +, approaches very closely the motion at aphelion of Mercury, which in this out of order arrangement will occupy the position *c*.[102] Then all these are related to each other in the same tuning. However, the motion of Venus at perihelion[103] divides the scale in one way, along with the three (or five) earlier ones, in the same harmonic kind, that is to say as the same planet's motion at aphelion, in its own tuning, meaning in the hard kind;[104] and the motion of the same Venus at perihelion divides the same scale in another way with the two later ones,[105] meaning not into different melodic intervals but rather into a different order for the melodic intervals, that is to say the order proper to the soft kind.

However, it is sufficient in this Chapter to have brought to your notice what the question is about; but why each feature has been carried out in this way, and the reasons not only for the agreements, but for the conflict in small details, will be made clear by the most lucid demonstrations in Chapter IX.

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[102] Taking 3 minutes 3 seconds to represent d_{ϱ}, the note *G*, a minor sixth below, corresponds to 183 seconds reduced in the ratio 5:8, which comes to 1 minute 54 seconds. This differs from the Earth's perihelion motion, 1 minute 55 seconds, by less than a comma. Kepler calculates the motion corresponding to a minor third below the perihelion motion of Venus, 3 minutes 3 seconds, to be 2 minutes 32 seconds. As the upper note is taken to be d_{ϱ}, the lower note is *c*. The aphelion motion of Mercury, 2 minutes 34 seconds, represents a note about a comma higher than *c*.

[103] This should be aphelion.

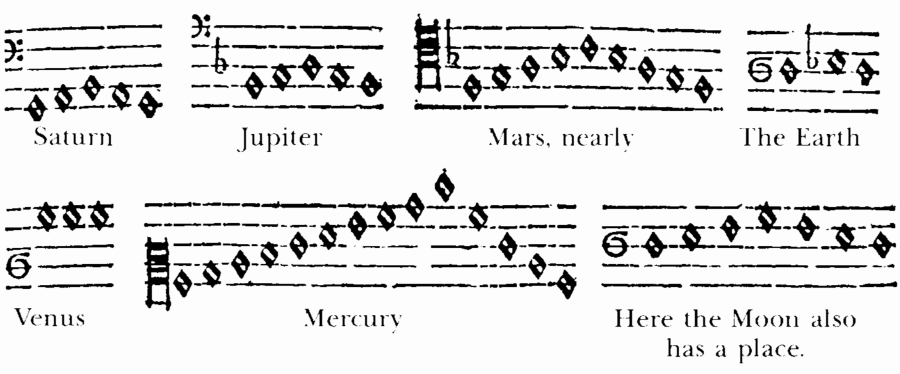
[104] The three (or five) earlier motions refer to the aphelion and perihelion motions of Saturn, the aphelion motions of the Earth and Jupiter and the perihelion motion of Mars, corresponding to the notes *G*, ♮ and *c*. Cf. note 98. As the aphelion motion of Venus corresponds to *e*, making an interval of a major sixth with *G*, all the notes are members of the hard scale.

[105] That is, the perihelion motion of the Earth and the aphelion motion of Mercury, corresponding to the notes *G* and *c*. As the perihelion motion of Mercury corresponds to d_{ϱ}, making an interval of a minor sixth with *G*, all the notes in this division belong to the soft scale.

Chapter VI.

That in the Extremes of the Planetary Motions Have Been Expressed, in a Fashion, the Musical Modes or Tones

This follows from what has already been said, and does not require many words; for the individual planets represent in a fashion individual positions in the system by their motion at perihelion, insofar as it is granted to each to traverse some particular interval in the musical scale, encompassed by certain notes in it, or positions in the system. Each starts from the note or position which belonged to its motion at aphelion in the previous Chapter, for Saturn indeed and the Earth *G*, but for Jupiter ♮, which can be transposed into the higher *G*, for Mars *f*ϱ, for Venus *e*; for Mercury *A*, in the higher system.[106] Here they are individually in the conventional notation. They do not indeed form the intermediate positions, which you here see filled in with notes, specifically, as they do the extremes; for they advance from one extreme to the opposite one not by leaps and intervals, but with a continually changing note, pervading all between (potentially infinite) in reality. I could not express that in any other way but by a continuous series of intermediate notes. Venus remains almost on unison, not amounting in the breadth of its tuning even to the smallest of the melodic intervals.[107]



[106] That is, in the case of Mercury, the range is described relative to the higher *G* of the soft scale, corresponding to the perihelion motion of Saturn. According to Kepler's diagram, Mercury ranges from *A* to *cc* in this transposed scale, which corresponds to the interval *c*ϱ to *ee* in the scale which takes the aphelion motion of Saturn as *G*. Mercury's tune is, of course, four octaves higher than the notes indicated here.

[107] The proportion of the motions of Venus represents an interval of nearly a diesis. In using Kepler's musical scales notation to compare the tunes of the planets, it should be noted that the tunes of Jupiter and Mercury are given in the transposed scale.

Yet by the designation of two notes in a common system, and the shaping of the skeleton of the octave, by spanning a definite melodic interval, there is a certain first step towards distinguishing tones or modes: therefore the musical modes have been distributed among the planets. To be sure I know that for the shaping and defining of distinct modes many things are needed, which are proper to human melody, that is to say when it has intervals; and so I have used the voice in a fashion.

Now it will be open to a musician to draw his own conclusion as to which mode each planet more nearly expresses, now that the extremes have here been assigned for him.[108] I should give to Saturn, among the conventional modes, the seventh or eighth, because if you set its tonic note as *G*, its motion at perihelion ascends to ♮; to Jupiter the first or second, because if its motion at aphelion is matched with *G*, its motion at perihelion reaches *b*; to Mars the fifth or sixth, not just because it almost attains a diapente, which is an interval common to all the modes, but chiefly because if it is reduced along with the rest to a common system, by its motion at perihelion it attains *c*, at aphelion it hints at *f*, which is the tonic of the fifth or sixth tone or mode. To the Earth I should give the third or fourth, because its motions revolve within a semitone; whereas for Mercury on account of the breadth of its interval all modes or tones will fit indifferently; for Venus, on account of the narrowness of its interval, none clearly, though because the system is common, the third and fourth fit it also, as relatively to the rest it occupies *e*.

The Earth sings MI FA MI, so that even from the syllable you may guess that in this home of ours MIsery and FAmine hold sway.

[108] See Book III, Chapter 14.

Chapter VII.

That the Universal Harmonies of all the Six Planets, as if in Common Counterpoint, Occur in Four Parts.

Now there is need, Urania,[109] of a grander sound, while I ascend by the harmonic stair of the celestial motions to higher things, where the true archetype of the fabric of the world is laid up and preserved. Follow me, modern musicians, and attribute it to your arts, unknown to antiquity: in these last centuries, Nature, always prodigal of herself, has at last brought forth, after an incubation of twice a thousand years, you, the first true offprints of the universal whole.[110] By your harmonizing of various voices, and through your ears, she has whispered of herself, as she is in her innermost bosom, to the human mind, most beloved daughter of God the Creator.

What harmonic proportions the pairs of neighboring planets embraced at the extremities of their motions has been expounded above. However, it happens very rarely that the two, especially the slowest, reach their extreme intervals at the same time. For example, the apsides of Saturn and Jupiter are about 81 degrees apart. Therefore, during the time in which this separation between them, in particular leaps of twenty years, measures out the whole Zodiac, eight hundred years slip by.[113] Yet the leap which concludes the eighth

Shall I be committing a crime if I demand some ingenious motet from individual composers of this age for this declaration? The royal psalter[111] and the other sacred books will be able to supply a suitable text for it. Yet take note that no more than six parts are in harmony in the heaven.[112] For the Moon warbles her solo independently, attending the Earth as at a cradle. Compare the symbols: in making the book, I promise to be a zealous overseer of the six parts. If anyone expresses more closely the heavenly music described in this work, to him Clio pledges a wreath, Urania pledges Venus as his bride.

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[109] The Muse of Astronomy.

[110] Kepler here refers to the more recent invention of polyphonic music, which he believed to be unknown to the Greeks.

[111] The Psalms of David.

[112] The number of voices in a motet was not restricted to six or less.

[113] Taking the periods of Saturn and Jupiter to be 30 years and 12 years respectively, the planets would return to the same relative position after 20 years. In this case, each leap would be 240° and the planets would return to the same position in the zodiac after three leaps or 60 years. With more accurate values of the periodic times, however, each leap is 243°, so that 40 leaps or 800 years are required for both planets to return to the same position in the zodiac. But on account of the nonuniformity of the motions of the two planets, the leaps are not exactly equal, so that a return to the same position in the zodiac cannot occur until the cumulative differences over 800 year intervals amount to one third of the circle; that is, half a single leap. For a leap (of 240°) from this point will represent a return to the original position. See Kepler's diagrams of the leaps in the *Mysterium cosmographicum*, Praefatio antiqua. Duncan (1981), 66.

century does not come precisely to the actual apsides; and if it should be a little further out of place, another eight hundred years must be awaited, to make it reasonably possible to look for a leap which will be more fortunate than the former one, and that must be repeated as often as the measure of the error is contained within the extent of half a single leap. The other individual pairs also produce such periods, though not so extensive. In the meantime, however, other harmonies of pairs of planets occur, which are not between motions which are both extreme; but with an intermediate motion, either one or both, and those at different tunings, so to speak. For as Saturn is extended from *G* to ♮ and a little further, and Jupiter from ♮ to *d* and beyond, therefore, the following harmonies at a diapason above can occur between Saturn and Jupiter: either of the thirds, and a diatessaron.[114] Of the thirds indeed, either one can occur through a tuning which covers the extent of the other; whereas the diatessaron can occur over the extent of a major tone.[115] For there will be a diatessaron not only from the *G* of Saturn to the *cc* of Jupiter but also from the *A* of Saturn to the *dd* of Jupiter, and over all the intermediate notes from the *G* and *A* of the former to the *cc* and *dd* of the latter.[116] However, the diapason and diapente occur only at the apsidal points. However, Mars which takes a larger interval of its own has acquired the feature that it also makes a diapason with the superior planets, by a certain latitude of tuning.[117] Mercury has occupied an interval of a size such that it generally sets up all harmonies with all planets, within one of its own periods, which is not more extensive than the space of three months. The Earth, on the contrary, and Venus much more, on account of the narrowness of their own intervals, restrict their harmonies not only with the other planets, but most of all their mutual harmonies with each other, to a remarkably small number. But if three planets must combine together into a single harmony, many alternations must be awaited. However, there are many harmonies, so that they occur all the more easily, when all the nearest planets catch their neighbors; and triple harmonies between Mars, the Earth and Mercury seem to occur rather often. However, harmonies of four planets now begin to be scattered over centuries, and those of five planets over myriads of years. However, an agreement together of all six is hedged about by very long gaps of ages; and I do not know whether it is altogether

[114] The intervals are above an octave because the motions of Jupiter were divided by 2 in order to bring them within the same octave as those of Saturn.

[115] The interval between the lowest note *G* of Saturn and the highest note *d* of Jupiter (neglecting the octave) is a fifth. Now the fifth is a combination of a major third and a minor third, and also of a fourth and a major tone.

[116] In designating the notes corresponding to the motions, Kepler here takes account of the difference of an octave.

[117] The notes *G* and *A* of Saturn in fact make a quadruple diapason with the g^3 and a^3 of Mars, and the c^1 of Jupiter makes a triple diapason with the c^4 of Mars.

impossible for it to occur twice, by a precise rotation, and it rather demonstrates that there was some beginning of time, from which every age of the world has descended.[118]

Character of the beginning of the world.

But if there could occur one single sixfold harmony, or one outstanding one among several, that undoubtedly could be taken as characterizing the Creation.

We must therefore enquire whether and in how many patterns altogether the motions of all the six planets are combined into one common harmony? The method of enquiry is to begin from the Earth and Venus, as these two do not make more than two consonances, and those (which contains the cause of this phenomenon) through very brief coincidences of the motions.

Come, therefore, let us first set up two, so to speak, skeletons of the harmonies, individually bounded by the pairs of extreme numbers (by which the terms of the tunings are represented), and let us set out to find what agrees with them out of the variety of motions permitted to each planet. Let the first skeleton have as the proportion between the Earth and Venus 3:5, and at the lowest tuning the daily motion of the Earth at aphelion, 57 minutes 3 seconds, at the highest tuning the motion of Venus at perihelion, 97 minutes 37 seconds.[119]

Therefore, the rest will be as follows. [See page 444.]

Therefore, the second skeleton will be one in which between the Earth and Venus there is another possible harmony, 5:8. In this case of the daily motion of Venus at aphelion, 94 minutes 50 seconds, one eighth part, 11 minutes 51 seconds +, taken five times, matches the motion of the Earth, 59 minutes 16 seconds, and a similar fraction of the motion of Venus at perihelion, 97 minutes 37 seconds, matches the motion of the Earth, 61 minutes 1 second. Therefore, the rest of the planets are in harmony with the following daily motions. [See page 445.]

Therefore, the experience of astronomy bears witness that universal harmonies of all the motions can occur, and both kinds, hard and soft; and in each kind in twofold shape or (if the term is permissible) tone; and in any one of four situations, with some latitude of tuning, and also with some variety in the particular harmonies of Saturn, Mars, and Mercury, each with the rest. Nor is it evinced only in the intermediate motions, but in absolutely all the extreme motions, except

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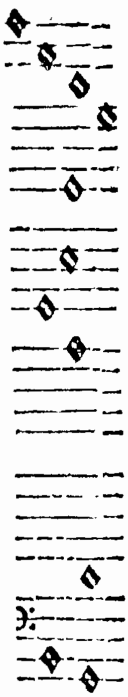
[118] Kepler had previously discussed the beginning of the world in the *Mysterium cosmographicum* (1596) and he commented on this in the notes to the second edition (1621). See Duncan (1981), 221–225.

[119] The ratio of the apparent daily motion of the Earth at aphelion and Venus at perihelion is not exactly 3:5. For the deepest tuning, Kepler takes the note g^4 of the Earth to represent its daily motion of 57′ 3″, in which case the note e^5 of Venus would correspond to a daily motion of 95′ 5″, slightly less than the true value. For the sharpest tuning, on the other hand, taking the note e^5 of Venus to represent the daily motion of 97′ 37″, the note g^4 of the Earth would correspond to a daily motion of 58′ 34″, slightly greater than the true value.

Harmonies of All the Planets or Universal Harmonies of the Hard Kind[120]

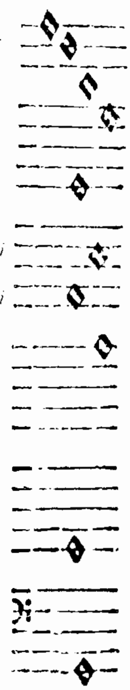
For ♮ to be in harmony



Planet	Note	At lowest tuning. Min. Sec.	At highest. Min. Sec.
Mercury	*e vii*	380. 20.	
	♮ *vii*	285. 15.	292. 48.
	g vi	228. 12.	234. 16.
	e vi	190. 10.	195. 14.
Venus	*e v*	95. 5.	97. 37.
Earth	*g iiii*	57. 3.	58. 34.
Mars	♮ *iii*	35. 39.	36. 36.
	g iii	28. 32.	29. 17.
Jupiter	♮ *i*		4. 34.
Saturn	♮	2. 14.	
	G	1. 47.	1. 49.

With this universal harmony Saturn coincides in its motion at aphelion, the Earth at aphelion, Venus at aphelion nearly; in the sharpest tuning Venus coincides nearly at perihelion; in the intermediate tuning Saturn coincides at perihelion, Jupiter at aphelion, Mercury at perihelion. Thus Saturn can coincide in two motions, Mars in two, Mercury in four.

For c to be in harmony



Planet	Note	At lowest tuning. Min. Sec.	At highest. Min. Sec.
Mercury	*e vii*	380. 20.	
	c vii	304. 16.	312. 21.
	g vi	228. 12.	234. 16.
	e vi	190. 10.	195. 14.
Venus	*e v*	95. 5.	97. 37.
Earth	*g iiii*	57. 3.	58. 34.
Mars	*c iiii*	38. 2.	39. 3.
	g iii	28. 32.	29. 17.
Jupiter	*c i*	4. 45.	4. 53.
Saturn	*G*	1. 47.	1. 49.

Here, while the rest remain the same, the motions of Saturn at perihelion and of Jupiter at aphelion are not included; but instead of them Mars coincides in its motion at perihelion.

The rest coincide in single motions: Mars alone in two, Mercury in four.

[120] The first chord, consisting of the notes *G*, ♮, and *e*, is that of *E* minor $^6_3$. Saturn can take the notes *G* and *h*, Mars also *g* and *h*, while Mercury can take all three at some point or other in its orbit. The remaining planets, Jupiter, Earth, and Venus, can take the notes *h*, *g*, and *e* respectively. The second chord, consisting of the notes *G*, *c*, and *e*, is that of *C* major $^6_4$. In this case, Saturn can take *G*, Jupiter *c*, Mars *c*, and *g*, the Earth *g*, Venus *e*, and Mercury all three. Kepler's description of these chords as of the hard kind shows that his classification hard and soft does not correspond to the modern major and minor. It seems that he is using the terms hard and soft in their original sense to mean any scale or chord that contains ♮ (*B* natural) or *b* (*B* flat) respectively.

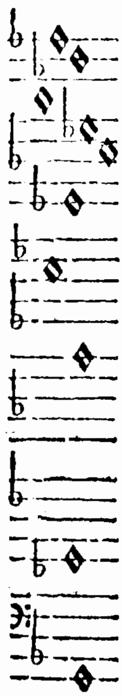
Harmonies of All the Planets or Universal Harmonies of the Soft Kind[121]

For ♮ to be in harmony[122]

		At lowest tuning. Min. Sec.	At highest. Min. Sec.
Mercury	*dϱ vii*	379. 20.	
	b vii	284. 32.	292. 56.
	g vi	237. 4.	244. 4.
	dϱ vi	189. 40.	195. 14.
Venus	*dϱ v*	94. 50.	97. 37.
Earth	*g iiii*	59. 16.	61. 1.
Mars	*b iiii*	35. 35.	36. 37.
	g iii	29. 38.	30. 31.
Jupiter	*b i*		4. 35.
Saturn	*b*	2. 13.	
	G	1. 51.	1. 55.

Here again at the intermediate tuning Saturn coincides in its motion at perihelion, Jupiter at aphelion, Mercury at perihelion. At the highest tuning the motion of the Earth at perihelion nearly coincides.

For c to be in harmony



		At lowest tuning. Min. Sec.	At highest. Min. Sec.
Mercury	*dϱ vii*	379. 20.	
	c vii	316. 5.	325. 26.
	g vi	237. 4.	244. 4.
	dϱ vi	189. 40.	195. 14.
	c vi		162. 43.
Venus	*dϱ v*	94. 50.	97. 37.
Earth	*g iiii*	59. 16.	61. 1.
Mars	*g iii*	29. 38.	30. 31.
Jupiter	*c i*	4. 56.	5. 5.
Saturn	*G*	1. 51.	1. 55.

Here also the motions of Jupiter at aphelion and of Saturn at perihelion are deleted, and that of Mercury at aphelion is admitted, nearly, in addition to its motion at perihelion. The rest remain the same.

[121] The first chord, consisting of *G*, *b*, and *dϱ* (*e* flat), is that of *E* flat major $^6_4$. The second chord, consisting of *G*, *c*, and *dϱ*, is that of *C* minor $^6_4$. It will be noted that two of Kepler's four harmonies of all the planets are second inversions or $^6_4$ chords. These chords have an interval of a fourth between the lowest notes. Although the fourth had been regarded as a consonance from antiquity onwards, from the beginning of the sixteenth century most composers had treated the $^6_4$ chord as if it were a dissonance which must be prepared by a suspension and then resolved by a downward step. A minority of theorists defended the use of the $^6_4$ chord as a consonance. Among them was Zarlino, but he only accepted the chord with a major third above the fourth, dismissing the chord with a minor third as dissonant in practice. Kepler was aware of the problem, as he refers to it in a letter to Maestlin written in 1599 (KGW 14, pp. 52–53). Yet in his account of the harmonies of all the planets, he is strangely content to accept tacitly the consonance of both kinds of $^6_4$ chord. Of course, he had demonstrated from geometry that the fourth is a consonance, but his usual insistence that theory must agree with the evidence of the ear is lacking in this instance. See Walker (1978), pp. 71–76 and Dickreiter (1973), pp. 111–112.

[122] This should be "for *b* to be in harmony."

for those of Mars at aphelion and of Jupiter at perihelion; for since the former occupies *f*ϱ and the latter *d*, Venus which perpetually occupies the intermediate *d*ϱ or *e* does not allow those dissonant neighbors in the universal harmony, which it would do if it had been granted room to go beyond *e* or *d*ϱ. This is the impediment which the marriage of the Earth and Venus has, as male and female, and they are the two planets which distinguish the kinds of harmonies, that is to say into hard and masculine, and soft and feminine. It is as if one of the spouses has done a favor to the other, that is to say the Earth is either at his aphelion, preserving, so to speak, his marital authority, and pressing on with tasks which are worthy of a man, pushing aside and banishing Venus to her perihelion as if to her distaff; or he has courteously allowed her to ascend towards her aphelion, or the Earth himself has descended towards his perihelion in the direction of Venus, and of her embraces, so to speak, so as to make love, laying aside for a little while his shield and arms, and those tasks which are proper for a man; for then the harmony is soft.

But if we command this antagonistic lady, Venus, to be silent, that is, if we consider what harmonies there can be, not of all the planets, but at least of the remaining five, excluding the motion of Venus, the Earth indeed is still astray on his note *g*, and does not ascend more than a semitone from it. Therefore, *b*, ♮, *c*, *d*, *d*ϱ, and *e* can still agree with *g*, in which case Jupiter, as you see, representing the note *d* by its motion at perihelion, is admitted. The difficulty of the motion of Mars at aphelion therefore remains. For the motion of the Earth at aphelion, which occupies *g*, does not allow Mars on *f*ϱ, whereas at perihelion, as has been stated above in Chapter V, it shrinks from agreement with the motion of Mars at aphelion by about half a diesis. [See page 447.]

However, there can also be the following harmony of the four planets Saturn, Jupiter, Mars, and Mercury, which would also include the motion of Mars at aphelion; but it has no latitude of tuning.[123] [See chart, page 448.]

Therefore, the motions of the heavens are nothing but a kind of perennial harmony (in thought not in sound[124]) through dissonant tunings, like certain syncopations or cadences (by which men imitate those natural dissonances), and tending towards definite and prescribed resolutions, individual to the six terms (as with vocal parts) and marking and distinguishing by those notes the immensity of time. Thus it is no longer surprising that Man, aping his Creator, has at last found

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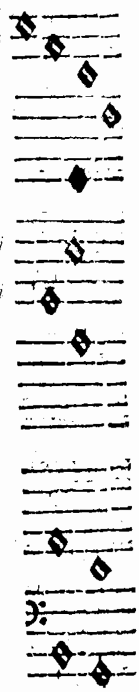
[123] The chords of the four planets Saturn, Jupiter, Mars, and Mercury, the one consisting of ♮, *d*, and *f*ϱ and the other of *A*, *d*, and *f*ϱ are ♮ minor $\frac{5}{3}$ and *A* major $\frac{5}{3}$ respectively.

[124] The celestial harmonies are here quite definitely described by Kepler as objects of rational thought. There is no hint of the idea of an instinctive reception of the harmonies that he had mentioned earlier. Cf. note 59.

Harmonies of Five Planets, Disregarding Venus[125]

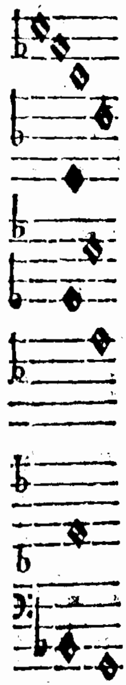
Of the Hard Kind



Planet	Note	At lowest tuning. Min. Sec.	At highest. Min. Sec.
Mercury	*d vii*	342. 18.	351. 24.
	♮ *vii*	285. 15.	292. 48.
	g vi	228. 12.	234. 16.
	d vi	171. 9.	175. 42.
Venus clashes	*e v*	95. 5.	97. 37.
Earth	*g iiii*	57. 3.	58. 34.
Mars	♮ *iiii*	35. 39.	36. 36.
	g iii	28. 31.	29. 17.
Jupiter	*d i*	5. 21.	5. 30.
	♮ *i*		4. 35.
Saturn	♮	2. 13.	
	G	1. 47.	

Here at the lowest tuning Saturn and the Earth coincide at their aphelia; at the intermediate tuning, Saturn at its perihelion, Jupiter at its aphelion; at the highest tuning, Jupiter at its perihelion.

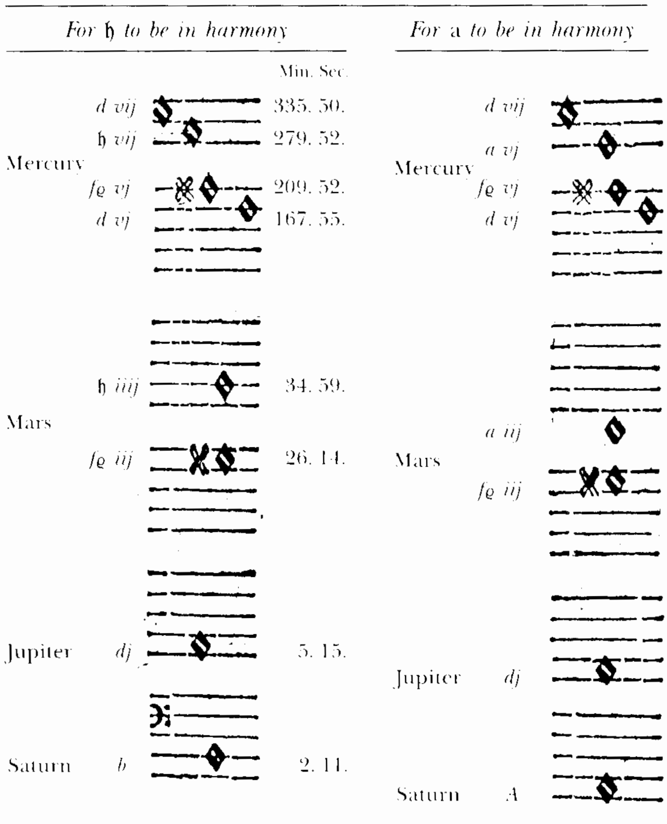
Of the Soft Kind



Planet	Note	At lowest tuning. Min. Sec.	At highest. Min. Sec.
Mercury	*d vii*	342. 18.	351. 24.
	♮ *vii*	273. 50.	280. 57.
	g vi	228. 12.	234. 16.
	d vi	171. 9.	175. 42.
Venus clashes	*e v*	95. 5.	97. 30.
Earth	*g iiii*	57. 3.	58. 34.
Mars	*b iiii*	34. 14.	35. 8.
	g iii	28. 31.	29. 17.
Jupiter	*d i*	5. 21.	5. 30.
Saturn	*b*	2. 8.	2. 12.
	G	1. 47.	1. 50.

Here Jupiter's motion at aphelion is not tolerated, but at the highest tuning Saturn very nearly coincides with it at perihelion.

[125] The chords of five planets (excluding Venus), the one consisting of *G*, ♮, and *d* and the other of *G*, *b*, and *d*, are *G* major $\frac{5}{3}$ and *G* minor $\frac{5}{3}$ respectively.



a method of singing in harmony which was unknown to the ancients, so that he might play, that is to say, the perpetuity of the whole of cosmic time in some brief fraction of an hour, by the artificial concert of several voices, and taste up to a point the satisfaction of God his Maker in His works by a most delightful sense of pleasure felt in this imitator of God, Music.

Chapter VIII.

Which in the Heavenly Harmonies Plays the Treble Part, Which Alto, Which Tenor, and Which Bass?

These are the names of human voices, and voices and sounds do not exist in the heaven, on account of the absolute quietness of the motions; but not even the phenomena in which we detect the harmonies are comprised in the category of true motion, since in fact we consider only the apparent motions seen from the Sun; and indeed there is no such cause in the heaven, calling for a definite number of voices for making harmony, as there is in human melody. In fact the number of the six planets going round the Sun came first, from the number of the five intervals, which were taken from the regular figures; and then afterwards (in the order of Nature not of time) the decision had to be made about the agreement of the motions. Nevertheless, for some unknown reason this wonderful agreement with human melody forces me so that I am compelled to pursue this part of the comparison also, even without solid natural cause. For those properties which in Book III, Chapter XVI, custom attributes, and nature appropriates to the bass, are the same as in a sense Saturn and Jupiter hold in the heaven; those of the tenor, we find in Mars; those of the alto belong to the Earth and Venus; and those of the treble are the same as Mercury has, if not in equality of distances, yet certainly in proportion. For whenever in the following Chapter the eccentricities of each planet are deduced from their own proper causes, and through them the proper intervals of the motions of each, that has a wonderful result, and I do not know whether it is not equally intended, and not merely an adjustment to necessities: that I. as bass is opposed to alto, so there are two planets which have the nature of the alto, and two of the bass, just as in any kind of music there is one voice on each side; and individual planets have the remaining individual voices. II. As the alto which is nearly the highest is in a narrow space, for necessary and natural reasons which were explained in Book III, so the planets which are nearly the inmost, the Earth and Venus, have very narrow distances between their motions, the Earth not more than a semitone, Venus not even a diesis.[126] III. And as the tenor is free, yet proceeds moderately, so Mars, with the sole exception of Mercury, can make the great-

[126] Although Kepler's theory of the narrow range of the alto fitted the restricted range of motions of the Earth and Venus, it was not in agreement with the views of the music theorists and composers of Kepler's time. Cf. Book III, note 197.

est distance, that is to say a diapente. IV. And as the bass makes harmonic leaps, so Saturn and Jupiter cover harmonic intervals, and have a distance between each other varying from a diapason to a diapente above the diapason. V. And as the treble is most free, more than all the rest, and the same is also the fastest, so also Mercury can range over more than a diapason and back again very quickly. But let this indeed be by the way: let us now hear the causes of the eccentricities.

CHAPTER IX.

The Origin of the Eccentricities of the Individual Planets in the Arranging of the Harmonies Between their Motions.

We see, therefore, that universal harmonies of all the six planets cannot be the result of accident, above all in the extremities of their motions, which as we have seen all coincided in universal harmonies, except for two, which coincided with the nearest harmonies to the universal ones. Also it is much less possible for it to come about accidentally that all the positions in the system of an octave established in Book III by the harmonic sections should be represented by extremes of the motions of the planets; and least likely of all that the very subtle business of the distinction of the heavenly harmonies into the two kinds, hard and soft, should happen by chance, without the care of a unique Craftsman. It therefore follows that the Creator, the fountain of all wisdom, the constant advocate of order, the eternal and transcendental wellspring of geometry and harmony, that He, I say, the very Craftsman of that which is in the heavens, has linked the harmonic proportions, which arose from the regular plane figures, to the five regular solid figures, and shaped from both classes the one most perfect archetype of the heavens. In it, just as through the five solid figures there shone forth the ideas of the spheres on which the six stars travel, so it was also through the offspring of the plane figures, the harmonies (deduced from them in Book III), that the measures of the eccentricities of the individual orbits, for proportioning the motions of the bodies, were given their terms. From these two things a single harmonization came about, and the greater proportions of the orbits yielded nothing to the lesser proportions of the eccentricities, which were necessary for the arranging of the harmonies; and correspondingly, of the harmonic proportions, those which had the greater affinity with each solid figure were chiefly fitted to the planets. Thus that could come about through the harmonies; and by that logic eventually both the proportions of the orbits, and their individual eccentricities, issued simultaneously from the archetype, whereas the individual periodic times resulted from the breadth of the orbits and the bulk of the bodies.[127]

While I strive to bring forth this line of argument into the light

[127] In fact, as Kepler explained in the *Epitome astronomiae Copernicanae*, four factors contributed to the periodic times. These were the size of the orbit, the strength of the solar force, the bulk (mass) of the planet, and its volume. Cf. note 46.

of human understanding by the conventional procedure of geometry, may the author of the heavens himself, the father of understanding, the bestower of mortal senses, Himself immortal and blessed above all, look favorably upon us, and prevent the darkness of our mind from putting forth anything concerning this His work which is unworthy of His majesty, and bring about that we as imitators of God may emulate the perfection of His works, by sanctity of life, for which He has chosen his Church in the lands, and cleansed it of sins by the blood of His son, with the help of the Holy Spirit, and may keep far from us all the dissonances of enmity, all contention, rivalry, anger, quarrels, dissension, sectarianism, envy, provocation, irritating facetiousness, and other works of the flesh. All who have the spirit of Christ will not only share my wish for these things, but will also strive to express them in deeds and to affirm their vocation, spurning all vicious practices of all factions though cloaked and painted over with an outward show of zeal, or of love of truth, or of singular erudition, or of deference to contentious teachers, or any other specious pretext. Holy Father, keep us in the concord of mutual love, so that we may be one, as You are one with Your Son, our Lord, and the Holy Spirit, and as You have made all Your works one by the delightful bonds of consonances; and so that from the restored concord of Your people the body of Your church may be built on this Earth just as You have constructed the heaven itself from harmonies.

A Priori Arguments[128]

I. Axiom

It is fitting that in any place whatever where it could be so, between the extreme motions both of individual planets and of pairs, harmonies ought to have been established of all kinds, so that such variety should adorn the world.

II. Axiom

The five intervals between the six spheres should have corresponded in size to a certain extent with the proportion of the geometrical spheres which are inscribed in and circumscribed about the five regular solid figures, and that in the same order which is natural to the figures themselves.[129]

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[128] These *a priori* demonstrations of the origins of the eccentricities follow the pattern of the similar demonstrations at the beginning of Book III (Chapters 1 and 2) of the origins of the harmonic proportions and in Book IV (Chapter 5) of the causes of the influential configurations or aspects. Again the axioms are to be regarded as working hypotheses, which Kepler believes to be true and wishes to justify if possible.

[129] These two axioms in effect postulate the archetype, while the references give the locations of a partial justification.

On this see Chapter I, and the Mysterium Cosmographicum (The Secret of the Universe) and the Epitome of Astronomy, Book IV.

III. Proposition

The distances between the Earth and Mars, and the same and Venus, should have been the smallest in proportion to their spheres, and very nearly equal; those between Saturn and Jupiter and between Venus and Mercury, intermediate and again nearly equal; and that between Jupiter and Mars the greatest.[130]

For by II, planets which correspond in position with figures which produce the smallest proportion in their geometrical spheres must similarly make the smallest proportion; those corresponding with figures of intermediate proportion produce an intermediate proportion; those corresponding with a figure of the greatest proportion produce the greatest proportion. But the order which applies between the figures of the dodecahedron and icosahedron also applies between the pairs of planets, one of Mars and the Earth, and the other of the Earth and Venus; and the order which applies to the cube and octahedron also applies to the pair of Saturn and Jupiter and the pair of Venus and Mercury; and last, the order which applies to the Tetrahedron also applies to the pair of Jupiter and Mars—see Chapter III. Therefore, the smallest proportion will be between the planetary spheres first named; but between Saturn and Jupiter a proportion almost equal to that between Venus and Mercury; and last, the greatest between the spheres of Jupiter and Mars.

IV. Axiom

All the planets ought to have their eccentricities, no less than motion in latitude, and also distances from the Sun, the fount of motion, varying according to the eccentricities.

Just as the essence of motion consists not in BEING, but in BECOMING, so also the appearance or shape of the region which a given planet would pass through in its motion does not BECOME solid straight away at the beginning, but by the passage of time in the end acquires not only its length but also its breadth and depth, making a complete threefold of dimensions; and thus it comes about gradually by the linking and accumulation of a great many revolutions that a kind of concave sphere is displayed, having the same center as the Sun, just as by a great many circles of a silken thread, linked with each other and wound together, the dwelling of a silkworm is made.

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[130] In the *Mysterium cosmographicum*, Kepler first ordered the solids by comparing the differences between the radii of their circumscribed and inscribed spheres with the intervals between the planets. He then gave *a priori* reasons for the order thus indicated by the data.

V. Proposition

To each pair of neighboring planets two different harmonies had to be attributed.

For by IV each planet has a longest separation from the Sun, and a shortest. Hence by Chapter III of this Book it will also have a slowest and a fastest motion. Therefore, there are two primary comparisons of extreme motions, one of the diverging motions of the two planets, and one of the converging. Now they must necessarily be different from each other, because the proportion of the diverging motions will be greater, that of the converging motions smaller. But there also had to be different harmonies between different pairs of planets, so that this variety might assist the adornment of the world, by Axiom I and also because there are different proportions between the distances, by III. But to each proportion of the spheres there correspond particular harmonic proportions, by their quantitative relationship, as was shown in Chapter V of this Book.

VI. Proposition

The two smallest harmonies, 4:5 and 5:6 have no place among the pairs of planets.

For 5:4 is as 1000:800, and 6:5 as 1000:833 +. But the spheres circumscribed round the dodecahedron and icosahedron have a greater proportion to those inscribed, that is to say a proportion of 1000:795, and so on, and these two proportions mark the distances between the planetary spheres which are closest to each other, or the smallest intervals: for in the other regular figures the spheres are further away from each other. However, in this case the proportion of the motions is still greater than the proportions of the distances, unless the proportion of the eccentricities to the spheres were vast, by Number 13 of Chapter III.[131] *Therefore, the smallest proportion of the motions is greater than 4:5 and 5:6. Therefore, these harmonies are barred in fact by the regular figures and are granted no place among the planets.*

VII. Proposition

The harmony of a diatessaron can have no place among the converging motions of pairs of planets, unless the combined proportions of their own extreme motions were more than a diapente.

For let the proportion be 3:4 between the convergent motions; and let there be, first, no eccentricity, and no proportion of their own between motions of in-

[131] See note 50. For the convergent motions to represent a small musical interval, the planets must be quite close. However, the interpolation of the five regular solids between the planetary spheres sets a lower limit to the closeness of neighboring planets. In the cases of the icosahedron and dodecahedron, the ratio of the radii of circumscribed and inscribed spheres is a minimum; namely about 1000:795. Kepler shows that this proportion is too large for the convergent motions to produce a major or a minor third.

dividual planets, but the same motions both convergent and mean. Then it follows that the corresponding distances, which on this hypothesis will be the radii of the spheres, constitute the square of the cube root of the proportion, that is to say 4480:5424, by Chapter III. But this proportion is already less than the proportion of the spheres of any of the regular figures.[132] *Thus the whole interior sphere would be cut by the faces of the regular figure inscribed in any exterior sphere. This, however, is contrary to Axiom II.*

Second, let the product of their own proportions between extreme motions be some definite amount; and let the proportion of the converging motions be 3:4 or 75:100, but the proportion of the corresponding distances be 1000:795, since no regular figure produces a smaller proportion between its spheres. And because the former proportion, that of the motions, inverted exceeds the latter, that of the distances, by a factor of 750:795, therefore let this factor be also divided into the proportion 1000:795 in accordance with the principle of Chapter III. The quotient is 9434:7950, the square root of the proportion of the spheres. Then the square of this, that is 8901:6320, or 10000:7100, is the proportion of the spheres.[133] *Divide this by the proportion of the converging distances, 1000:795. The quotient will be 7100:7950, about a major tone. This should be as a minimum the product of the two proportions which the mean distances have to the converging distances on either side, for a diatessaron to be possible between the converging motions. Therefore, the product of the proportions produced by the divergent extreme distances to the convergent extreme distances is about the square of that, that is two tones; and the product for their own motions is again the square of that, that is four tones, which is more than a diapente. Therefore, if for two neighboring planets the product of their own motions is less than a diapente, a diatessaron between their converging motions will not be possible.*

VIII. Proposition

Saturn and Jupiter ought to have had the harmonies 1:2 and 1:3, that is a diapason and a diapente above the diapason.

For they are themselves the first and highest of the planets, and have got

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[132] By the third (harmonic) law, $r/R = (3/4)^{2/3} = 0.826$. This is greater than 0.795. Kepler, however, regards 0.826 as the smaller proportion as it is closer to 1.

[133] The inverse proportion of the motions $m_a{:}M_p = 4{:}3$ exceeds the proportion of the distances $R_p{:}r_a = 1000{:}795$ by the proportion 795:750, so that $(m_a/M_p)\cdot(r_a/R_p) = 795/750$. Taking this factor out of $R_p/r_a = 1000/795$, we have $(R_p/r_a)^2\cdot(M_p/m_a) = 9434/7950$. This has been shown to be $(R/r)^{1/2}$. (See note 50). Hence $R{:}r = 10000{:}7100$. This exceeds the proportion of the convergent distances $R_p{:}r_a$ by the proportion 7950:7100, corresponding approximately to a major whole tone; that is, $(r_a/R_p)\cdot(R/r)$ represents approximately a major whole tone. For small eccentricities, we can take $(R_aR_p)/(r_ar_p) = R^2/r^2$, so that $(r_a/r_p)\cdot(R_a/l_p) = (r_aR)^2/(R_pr)^2$, corresponding to two major whole tones. Now $m_p/m_a = r_a^2/r_p^2$ and $M_p/M_a = R_a^2/R_p^2$ (see note 41), so that $(m_p/m_a)\cdot(M_p/M_a) = (r_a/r_p)^2\cdot(R_a/R_p)^2$, corresponding to four major whole tones, which is greater than a fifth. In other words, for the convergent motions to correspond to a fourth, the combination of the harmonies of the single planets must be greater than a fifth.

the first of the figures, the cube, by Chapter I of this Book; and these harmonies are the first in the order of nature and are the heads of the first families among the figures, the Bisecting or Tetragonic, and the Trigonic, by what has been said in Book I. However, that which is the head, the diapason, 1:2 is very slightly greater than the semitriple[134] *of the proportion of the spheres of the cube. Thus it is appropriate for it to become the lesser proportion of the motions of the cubic planets, by Chapter III, Number 13; and in consequence, 1:3 serves as the greater proportion. However, the same conclusion is also reached as follows For if some harmony is to some proportion found between the spheres of the figures as the proportion of the apparent motions, as seen from the Sun, to the proportion of the mean distances, such a harmony will deservedly be attributed to the motions. But it is natural that the proportion of the diverging motions should be much greater than the sesquialterate proportion of the spheres, by the end of Chapter III. That is, it approaches the square of the proportion of the spheres; and 1:3 is also the square of the proportion of the cubic spheres, namely, as we say the semitriple. Then the threefold harmony ought to belong to the divergent motions of Saturn and Jupiter. See the numerous other affinities of these proportions with the cube above in Chapter II.*

IX. Proposition

Saturn's and Jupiter's extreme motions' own proportions combined ought to have come to 2:3, about a diapente.

That follows from the foregoing proposition; for if the motion of Jupiter at perihelion is triple that of Saturn at aphelion, and on the other hand the motion of Jupiter at aphelion is double that of Saturn at perihelion, then on dividing 1:2 into 1:3, the quotient is 2:3.

X. Axiom

When there is a free choice among the others, the superior planet ought to have as its own proportion in its motions that which is prior by nature, or that which is of the more distinguished kind, or even that which is greater.[135]

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[134] That is, the proportion $1:\sqrt{3}$. The proportion of the diapason, 1:2, is slightly greater, because $1/\sqrt{3}$ is closer to 1 than is 1/2.

[135] In fitting the regular polyhedra between the planetary spheres, Kepler started with the superior planets. He explained that, because the region of the fixed stars was the most important part of the universe outside the earth, the cube, which was the first in the class of primary solids, should be closest to the sphere of fixed stars and establish the first proportion of the distances: that is, the distances of Saturn and Jupiter. *Mysterium cosmographicum*, Chapter 5. See Duncan (1981), 109. The natural order of the planets is thus established as proceeding downwards. Kepler needs the axiom in the next proposition to decide which planet should have the major third and which the minor third.

XI. Proposition

The proportion of the motion of Saturn at aphelion to that at perihelion ought to have been 4:5, a major third, but that of Jupiter's motions 5:6, a minor third.

For because in combination they hold 2:3, but this is not divided harmonically except into 4:5 and 5:6; therefore God the Governor-General has divided the harmony 2:3 harmonically, by Axiom I, and has given the harmonic part of it, which is the greater, and of the more distinguished hard kind, in fact masculine, to the greater and higher planet Saturn, and the lesser, 5:6, to the lower, Jupiter, by X.

XII. Proposition

Venus and Mercury ought to have had the major harmony 1:4, the double diapason.

For just as the cube is the first figure of the primaries, so the octahedron is the first of the secondaries, by Chapter I of this Book. And just as the cube, considered geometrically, is the outer, and the octahedron the inner, that is to say the latter may be inscribed in the former, so also in the world Saturn indeed and Jupiter are the beginning of the higher and outer planets, or on the outside, whereas Mercury and Venus are the beginning of the inner planets, or on the inside; and interposed between their courses is the octahedron: see Chapter III. Of the harmonies therefore Venus and Mercury ought also to have one which is primary and akin to the octahedron. Furthermore, among the harmonies after 1:2 and 1:3 there follows in the natural order 1:4, and it is akin to the cubic 1:2 because it has arisen from the same group of figures, that is the tetragonic, and is commensurable with it, that is the square of it; whereas the octahedron is akin to the cube and commensurable with it. Also 1:4 is akin to the octahedron, independently, on account of the quaternary number which is indeed within it, and in fact hidden in the octahedron the quadrangular figure, the proportion of the spheres of which is stated as semiduple. Therefore, the harmony of its proportion is 1:4, which is by continuous multiplication in the proportion of squares, that is to say the fourth power of the semiduple: see Chapter II.[136] *Then Venus and Mercury ought to have had 1:4. And because 1:2 in the cube is the lesser harmony between the two, since the outermost location has fallen to it, there will be in the octahedron this proportion 1:4 which is now the greater harmony between the two, as it is the one to which the innermost position has fallen. But the following is also a reason why 1:4 has here been given as the greater, not the lesser. For since the proportion of the spheres of the octahedron is semitriple, assuming that the inscription of the octahedron between the planets is perfect (though it is not perfect, but penetrates Mercury to a certain extent, which is in our favor), therefore the proportion of the converging motions must*

Remember that the words square, cube, semitriple, and so on sometimes give their name to individual proportions, considered absolutely as a kind of quantity; but they sometimes express the proportion of pairs of proportions compared with each other.

[136] The semiduple proportion is $1:\sqrt{2}$ and the fourth power of the semiduple proportion is $1:(\sqrt{2})^4 = 1:4$.

be smaller than the sesquialterate of that semitriple proportion. But even 1:3 is plainly the square of the semitriple, and thus greater than the correct proportion by the amount by which 1:4, which of course is greater than 1:3, is greater than the correct amount. Then not even the square root of 1:4 is tolerated between the converging motions.[137] *Therefore 1:4 cannot be the smaller octahedric proportion; therefore it will be the greater. Further, 1:4 is akin to the octahedric square, the proportion of the spheres of which is semiduple, in the same way as 1:3 is akin to the cube, as the proportion of its spheres is semitriple. For just as 1:3 is a power of the semitriple, that is to say its square, so also this 1:4 is a power of the semiduple, that is to say the square of its square, that is its fourth power. Hence if 1:3 should have been the greater harmony of the cube, by VIII, therefore 1:4 ought also to be the greater harmony of its octahedron.*

XIII. Proposition

The extreme motions of Jupiter and Mars ought to have had as their harmonies, for the greater in fact 1:8, about a triple diapason, and as the lesser 5:24, a minor third above the double diapason.

For because the cube has been allotted 1:2 and 1:3, but the proportion of the spheres of the tetrahedron, which is located between Jupiter and Mars, designated as the triple, is the square of the proportion of the spheres of the cube, designated as semitriple, then it was appropriate that proportions for the motions should also be fitted to the tetrahedron which are the square of the cubic proportions. Now the proportions which are the squares of 1:2 and 1:3 are in fact 1:4 and 1:9. But 1:9 is not harmonic, and 1:4 has already been taken up for the octahedron. Therefore, the neighboring harmonies to these proportions had to be taken, by Axiom I. Now first of all the ones which are neighbors to it are 1:8 as the smaller and 1:10 as the greater. Between these the choice is made by affinity with the tetrahedron, which has nothing in common with the pentagon, since 1:10 is of the pentagonic group. However, the tetrahedron's greater affinity is with 1:8, on many accounts, which are to be found in Chapter II. Further, on 1:8, another feature is that just as 1:3 is the greater proportion of the cube, and 1:4 the greater of the octahedron, because they are powers of the proportions between the spheres of the figures, so also 1:8 should have been the greater proportion of the tetrahedron. For as its body[138] *is twice that of the octahedron inscribed in it, as stated in Chapter I, so also the term 8 of this*

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[137] The proportion of the convergent motions is smaller than $1:(\sqrt{3})^{3/2} = 1:2.28$ approximately. Hence 1:3 is greater than the proportion of the convergent motions and, according to Kepler, 1:4 is too large by as much again. The proportion 1:3 is greater than the true proportion of the convergent motions in the ratio 3:2.28 = 1.32:1. The proportion 1:4 is greater in the ratio 4:2.28 = 1.75:1 = 1.32^2:1. It follows that 1:2 is too great in the proportion 1.32:1. This would make the true proportion 0.66:1, which may be compared with the proportion 3:5 of the major sixth. Here Kepler is concerned only to demonstrate that the interval of the convergent motions is smaller than an octave.

[138] That is, its volume.

tetrahedric proportion is twice the term 4 of the octahedric proportion. Further, since 1:2, the lesser proportion of the cube, is one diapason, and 1:4 the greater proportion of the octahedron is two diapasons, so in this instance 1:8, the greater proportion of the tetrahedron should have been three diapasons. Now, it had to have more diapasons than two, because since the lesser tetrahedric harmony must necessarily be the greatest of all the smaller harmonies in the other figures (inasmuch as the proportion of the tetrahedric spheres is the greatest of all those of the figures), the greater tetrahedric harmony also should have exceeded the greater harmonies of the others in its number of diapasons. Last, the threefold nature of the intervals in diapasons has kinship with the triangular type of the tetrahedron, and has a certain perfection in accordance with the universal perfection of the Trinity, since the eightfold also, its term, is the first of the cubic numbers, which are perfect in quantity, that is to say of three dimensions.

II. To 1:4, or 6:24, the neighboring harmonies are 5:24, in fact the greater, and the lesser 6:20 or 3:10. Again, however, 3:10 is of the pentagonic group, which has nothing in common with the tetrahedron. But 5:24 on account of the numbers 3 and 4 (of which the numbers 12 and 24 are offspring) has kinship with the tetrahedron. For we neglect the other lesser terms, that is to say 5 and 3, here, because their degree of affinity with the figures is the lowest, as may be seen in Chapter II. In addition, the proportion of the spheres of the tetrahedron is the triple; and the proportion of the converging distances ought also to be the same size, about, by Axiom II. But by Chapter III the proportion of the converging motions is approximately the inverse of the sesquialterate of that of the distances, whereas the sesquialterate of the triple proportion is about that between 1000 and 193.[139] *Therefore, in units in which the motion of Mars at aphelion is 1000, Jupiter will be a little greater than 193, much less than 333, a third part of 1000. Therefore, not the harmony 10:3, that is 1000:333, but the harmony 24:5, that is 1000:208, holds the place between the converging motions of Jupiter and Mars.*

XIV. Proposition

The extreme motions' own proportion in the case of Mars ought to have been greater than a diatessaron, 3:4, and about 18:25.

For let precisely the harmonies 5:24 and 1:8, or 3:24, be attributed in this case to Jupiter and Mars in common, by Proposition XIII. Divide the lesser, 5:24, into the greater, 3:24. The quotient is 3:5, the product of both planets' own proportions. But Jupiter's own proportion alone was found in fact in Proposition XI above to be 5:6. Divide that, therefore, into the product of their own motions, 3:5. That is, divide 25:30 into 18:30. The quotient is Mars' own proportion, 18:25, which is greater than 18:24 or 3:4. However it will become yet greater, if by the following arguments the greater common proportion 1:8 were to be increased.

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[139] The sesquialterate of the triple proportion is $1:3^{3/2} = 1:5.196 = 193:1000$ approximately. This is close to $208:1000 = 5:24$, corresponding to a minor third above a double diapason.

XV. Proposition

Between the converging motions of Mars and the Earth, of the Earth and Venus, and of Venus and Mercury, the harmonies 2:3, the diapente, 5:8, the soft sixth, and 3:5, the hard sixth, had to be shared; and in that order.

For the dodecahedron and icosahedron, the figures interposed between Mars, the Earth and Venus, have the smallest proportion between their spheres, circumscribed and inscribed. Then they ought to have the smallest of the possible harmonies, being akin on that account, and so that Axiom II may have its place. But the smallest harmonies of all, that is 5:6 and 4:5, are not possible, by VI. Then the figures stated ought to have the harmonies next greater than those, that is either 3:4 or 2:3 or 5:8 or 3:5.

Again the figure interposed between Venus and Mercury, that is to say the octahedron, has the same proportion in its spheres as the cube. But to the cube as its lesser harmony, which is between its converging motions, belongs the diapason, by VIII. Then by analogy the octahedron ought to have had a proportion of the same size, that is 1:2, as its smaller one, if no diversity is included. However, diversity is included, to the extent that in the case of the cubic planets in fact, that is Saturn and Jupiter, their individual motions' own proportions combined produced a total not greater than 2:3. In this case, however, of the octahedric planets, Venus and Mercury, their individual motions' own proportions combined will make a total greater than 2:3, which is easily apparent in the following way. For suppose that what was required was the proportion between the cube and the octahedron, if it were the only one: let, I mean, the lesser octahedric proportion be greater than those which have been prescribed here, and let it be absolutely as great as was the cubic, that is to say 1:2, whereas the greater was 1:4 by XII. Therefore, if this is divided by the lesser proportion which we have just assumed, 1:2, there still remains 1:2 as the product of Venus and Mercury's own proportions. But 1:2 is more than the product of Saturn and Jupiter's own proportions, 2:3. And indeed the consequence of this greater product is a greater eccentricity, by Chapter III; while the consequence of this greater eccentricity is a lesser proportion between the converging motions, by the same Chapter III. Hence it comes about from the multiplication of this greater eccentricity by the proportion between the cube and octahedron that a lesser proportion than 1:2 is also required between the converging motions of Venus and Mercury. It was also appropriate for Axiom I that as the harmony of the diapason was taken up for the cubics, another which was very close should be adapted to the octahedrics, and by the previous proof, one less than 1:2. Now the proportion next smaller than that is 3:5, which, as it is the greater of the three, the figure with the greater proportion between its spheres ought to have had, that is to say the octahedron. Therefore, the lesser harmonies 5:8 and 2:3 and 3:4 were left for the icosahedron and dodecahedron, figures with a lesser proportion between their spheres.

Now these remaining harmonies were distributed among the two remaining figures. For just as of the figures, although they have equal proportions between their spheres, the harmony 1:2 has in fact been allotted to the cube, but the *smaller 3:5 to the octahedron, for the reason that the product of Venus' and Mercury's own proportions would exceed the product of Saturn's and Jupiter's own proportions, so also in this case the dodecahedron, even though it makes the same proportion between its spheres as the icosahedron, ought to have had a lesser harmony than the icosahedron, but the closest, on account of a similar reason, that is because the latter figure is between the Earth and Mars, the eccentricity of which had been made large among the superior planets; whereas the eccentricities of Venus and the Earth, as we shall hear in what follows, are the smallest. And since the octahedron has 3:5, the icosahedron, of which the spheres have a smaller proportion, has the next, a little smaller than 3:5, that is 5:8. Therefore, there was left for the dodecahedron either 2:3, which remained, or 3:4; but preferably the former, inasmuch as it is closer to the icosahedric 5:8, as their figures are also similar.*

But even 3:4 was not possible. For although among the superior planets Mars' extreme motions' own proportion was great enough, yet the Earth, as has already been said, and will be clear in what follows, contributed as its own a proportion too small for the product of the two to exceed a diapente. Therefore, 3:4 could not have the position, by VII; and all the more so because as will follow in Proposition XLVII the proportion of the converging distances should have been greater than 1000:795.

XVI. Proposition

The proportions of the motions of Venus and Mercury combined, their own in each case, should have amounted to about 5:12.

For divide the smaller harmonic proportion, 3:5, attributed in common to this pair, by Proposition XV, into the greater of them, 1:4 or 3:12, by XII. The quotient, 5:12, is the product of the two's own proportions. Thus of Mercury alone the extreme motions' own proportion is less than 5:12 by the amount of Venus' own proportion. This is to be understood from these primary arguments. For below by the secondary arguments, with the inclusion of the common harmonies of the two as a kind of yeast, it will turn out that Mercury's own proportion alone holds 5:12.

XVII. Proposition

The harmony of the divergent motions of Mars and the Earth could not have been less than 5:12.

For Mars alone in its motions' own proportion has got more than a diatessaron, and more than 18:25 by XIV. Now their lesser harmony is a diapente, 2:3, by XV. Therefore these two parts combined make 12:25. But the Earth also must have its own proportion, by Axiom IV. Then since the harmony of the divergent motions consists of the three elements stated, that will be greater than 12:25. But the harmony next greater than 12:25, or 60:125, is 5:12, that is to say 60:144. Hence if a harmony is needed for this greater proportion of the motions of the two planets, by Axiom I it cannot be less than 60:144 or 5:12.

Up to this point, therefore, all the other pairs of planets have been fitted to their pairs of harmonies by necessary arguments. Only the pair of the Earth and Venus so far has been allotted one harmony alone, 5:8, by the Axioms adopted until now. We shall therefore now search further for its other harmony, that is the greater, or that of the diverging motions, making a new start.

A Posteriori Arguments.[140]

XVIII. Axiom

The universal harmonies of the motions must have been established by the combination of six motions, especially through the extreme motions.

This is proved by Axiom I.

XIX. Axiom

The same universal harmonies must have occurred over a certain range of the motions, that is to say so that they should happen all the more often.

For if they had been confined to particular points in the motions it could have come about that they would never occur, or certainly very rarely.[141]

XX. Axiom

As the most natural distinction of the kinds of harmonies is into hard and soft, as has been proved in Book III, so universal harmonies of both kinds must have been arranged between the extreme motions of the planets.

XXI. Axiom

Different types of harmonies of both kinds must have been organized so that the beauty of the world might be expressed in harmony through all possible forms of variation, and that by the extreme motions, or at least some of them.

By Axiom I.

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[140] In this second set of axioms and propositions, besides resolving the remaining problem concerning the Earth and Venus and refining the theory of the harmonic intervals in the motions of the planets, Kepler sets out his *a priori* reasons for the celestial music in the form of chords and scales, which he has described in Chapters V to VII.

[141] This axiom is needed to justify the variation in tuning, described in Chapter VII, and the inclusion in the chords of all the planets of notes corresponding to positions other than aphelion and perihelion.

XXII. Proposition

The extreme motions of the planets must have represented positions or notes in the system of a diapason, or notes in the musical scale.

For the origin and comparison of harmonies which start from one common term generated the musical scale, or division of the diapason into its positions or notes, as proved in Book III. Therefore, since different harmonies between the extremes of the motions are required, by Axioms I, XX and XXI, hence in some system, heavenly or in the harmonic scale, a real differentiation between the extremes of the motions is required.

XXIII. Proposition

There must have been one pair of planets between the motions of which no harmonies could exist except the two sixths, the major, 3:5, and the minor, 5:8.

For since there was a necessary distinction between the kinds of harmonies, by Axiom XX, and that by the extremes of the motions at the apsides, by XXII, because only the extremes, that is to say the slowest motion and the fastest, need to be defined so as to arrange and order them, the intermediate tunings emerge of their own accord during the actual passage of the planet from its slowest motion to its fastest, without particular attention. Therefore, this ordering could not have come about except when a diesis, or 24:25, was marked out by the extremes of the two planetary motions, on account of the fact that the kinds of harmonies are distinguished by a diesis, from what has been explained in Book III. But a diesis is the difference either between the two thirds, 4:5 and 5:6, or between the two sixths, 3:5 and 5:8, or the same harmonies augmented by one or more intervals of a diapason. However, the two thirds, 4:5 and 5:6, had no place between pairs of planets, by Proposition VI; but neither had thirds or sixths augmented by the interval of a diapason been found anywhere, except for 5:12 in the pair of Mars and the Earth, and that only with its comrade 2:3. Thus the intermediate proportions 5:8 and 3:5 and 1:2 were equally admitted. Then it remains for the two sixths, 3:5 and 5:8, to be given to one pair of planets. But also the sixths alone had to be conceded to the variation of those motions, in such a way that they neither extended their terms to include the next greater interval of one octave, 1:2, nor contracted them to the narrowness of the next lesser of a diapente, 2:3. For although it is true that if two planets make a diapente with the converging extremes of their motions, and a diapason with the diverging, the same planets can also make sixths, and so can also traverse a diesis, yet that would not smack of the singular providence of the Orderer of the motions. For the diesis, the smallest of the intervals, which is potentially hidden in all the greater harmonies which are included between the extremes of the motions, would then in fact itself be traversed by the intermediate motions, which vary continuously with the tuning. Yet it is not marked out by their extremes, since the part is always less than the whole, that is the diesis than the greater interval 3:4, which is between 2:3 and 1:2, which would in this case be supposed to be wholly marked out by the extremes of the motions.

XXIV. Proposition

Two planets which change the kind of harmony ought to make the difference between their own proportions of the extremes of their motions a diesis, and one's own proportion ought to be greater than a diesis; and by their motions at aphelion they ought to make one of the sixths, by those at perihelion the other.

For since the extreme motions make two harmonies, differing by a single diesis, that can happen in three ways: for either the motion of one planet may remain constant, and that of the other vary over a diesis; or else both may vary over half a diesis and they make 3:5, a major sixth, when the superior planet is at aphelion, and the inferior one at perihelion, but making a deviation from those intervals, mutually going to meet each other, the superior up to perihelion, the inferior down to aphelion, they may make 5:8, a minor sixth; or last, one may vary its motion more than the other from aphelion to perihelion, and there may be a difference of one diesis, and so there may be a major sixth between the two at aphelion, a minor sixth between the two at perihelion. However, the first way is not legitimate, for one of these planets would have no eccentricity, contrary to Axiom IV. The second way would be less beautiful and less convenient: less beautiful, because less harmonic. For the two planets' motions' own proportions would have been unmelodic, as anything less than a diesis is unmelodic. It is better, however, for one planet alone to labor under this unmelodic littleness. Indeed, it could not even have come about, because on this basis the extreme motions would have strayed from the positions in the system, or notes in the musical scale, contrary to XXII. Also it would have been less convenient, because the sixths would have occurred only at those moments at which the planets would have been at opposite apsides: there would have been no range over which these sixths, and thus the universal harmonies resulting from them, could have occurred. Therefore, the universal harmonies, when all the positions of the planets had been brought back to the restrictions of definite and unique points on their orbits, would have been very rare, contrary to Axiom XIX. There remains therefore the third way, in which indeed each of the planets varies its own motion, but one more than the other by one perfect diesis at least.

XXV. Proposition

Of the planets which change the kind of harmony, the upper ought to have a proportion between its own motions less than a minor tone, 9:10, and the inferior less than a semitone, 15:16.

For either they will make 3:5 with their motions at aphelion, or at perihelion, as has already been stated. Not with those at perihelion; for in that case the proportion of the motions at aphelion would be 5:8. Therefore, the inferior planet would have one diesis more in its own proportion than the superior planet, by the same previous statement. However that is contrary to Axiom X. Therefore, they make 3:5 with their motions at aphelion, 5:8 at perihelion, less than in the former case by 24:25. But if the motions at aphelion make a hard sixth, 3:5, then the motion of the superior planet at aphelion will make with the motion *of the inferior at perihelion more than a hard sixth, for the inferior will add the whole of its own proportion. In the same way, if the motions at perihelion make a soft sixth, 5:8, the motion of the superior planet at perihelion and that of the inferior at aphelion will make less than a soft sixth; for the inferior takes away the whole of its own proportion. But if the inferior planet's own proportion equalled a semitone, 15:16, in that case as well as the sixths a diapente could also occur, because a soft sixth diminished by a semitone becomes a diapente: but that is contrary to Proposition XXIII. Therefore, the inferior planet has less than a semitone in its own interval. And because the superior planet's own proportion is greater than the inferior planet's own, by one diesis, whereas a diesis added to a semitone makes a minor tone, 9:10, therefore the superior planet's own proportion is less than a minor tone, 9:10.*

XXVI. Proposition

Of planets which change the kind of harmony, the superior ought to have had a double diesis, 576:625, that is nearly 12:13, for the interval between its extreme motions, or a semitone, 15:16, or something intermediate, separated from either the former or the latter by a comma, 80:81; whereas the inferior planet ought to have either a simple diesis, 24:25, or the difference between a semitone and a diesis, which is 125:128, that is nearly 42:43, or last, in a similar way something intermediate, separated by a comma, 80:81, either from the former or the latter, that is the former having a double diesis, the latter a simple diesis, both intervals diminished by a comma.

For the superior planet's own proportion ought to be greater than a diesis, by XXV, but less than a tone, 9:10, by the previous Proposition. But in fact the superior one ought to exceed the inferior by one diesis, by XXIV. And harmonic beauty urges that these planets' own proportions, if owing to their small size they cannot be harmonic, should at least be among the melodic, if that is possible, by Axiom I. But the only two melodic intervals smaller than a tone, 9:10, are the semitone and the diesis, and these differ from each other not by a diesis but by some smaller interval, 125:128. Therefore, the superior planet cannot have a semitone and the inferior a diesis, at the same time; but either the superior will have a semitone, 15:16, and the inferior 125:128, that is 42:43, or the inferior will have a diesis, 24:25, but the superior a double diesis, 12:13 nearly. But since the two planets have equal rights, therefore if the nature of melody had to be violated in their own proportions, it had to be violated equally in both cases, so that the difference between their own intervals could remain exactly a diesis, to differentiate the necessary kinds of harmonies, by XXIV. Now the nature of melody was equally violated in both cases if the factor by which the superior planet's own proportion fell short of a double diesis, or exceeded a semitone, was the factor by which the inferior's own proportion fell short of a simple diesis, or exceeded the interval 125:128.

Furthermore, this excess or shortfall should have been a comma, 80:81, because again no other interval was demonstrated from the harmonic proportions, and so that the comma should be expressed among the heavenly motions in the *same way as it was expressed in the harmonic proportions, that is to say only by the excess and shortfall of the intervals between each other. For among the harmonic intervals the comma is the distinction between the tones, major and minor, and is not noticed in any other way.*

It remains for us to investigate which intervals are to be preferred of those suggested, whether it should be the dieses, simple for the inferior planet, double for the superior, or rather a semitone for the superior, and 125:128 for the inferior. And the dieses have the winning arguments. For although the semitone has been expressed in various ways in the musical scale, yet its partner 125:128 has not been expressed. On the other hand, both the diesis has been expressed in various ways, and the double diesis in a way, that is in the resolution of tones into dieses, semitones and limmata; for in that case, as has been stated in Book III, Chapter VIII, two dieses follow next to each other in two positions. Another argument is that in making the proper distinction between the kinds the diesis has rights, the semitone none. Therefore, greater attention should have been paid to the dieses than to the semitone. The outcome of all this is the following: the superior planet's own proportion ought to be 2916:3125, or 14:15, nearly; the inferior's own proportion 243:250, or 35:36 nearly.

Do you ask whether the highest creative wisdom would have been taken up with searching out these thin little arguments? I answer that it is possible for many arguments to escape me. But if the nature of harmony has not supplied weightier arguments, that is in the case of proportions which descend below the size of all the melodic intervals, it is not absurd for God to have followed even these, however thin they may appear, since he has ordered nothing without reason. For it would be far more absurd to declare that God has snatched these quantities, which are in fact below the limit of a minor tone prescribed for them, accidentally. Nor is it sufficient to say that He adopted that size because that size pleased Him. For in matters of geometry which are subject to freedom of choice it has not pleased God to do anything without some geometrical reason or other, as is apparent in the borders of leaves, in the scales of fishes, in the hides of wild beasts, and in their spots and the ordering of their spots, and the like.[142]

XXVII. Proposition

The greater proportion of the motions of the Earth and Venus ought to have been a hard sixth between the motions at aphelion, and the smaller a soft sixth between the motions at perihelion.

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[142] This is the clearest statement of Kepler's belief in the universality of design in nature. When he wrote the *Mysterium cosmographicum*, it had been his intention to extend his geometrical theories to the explanation of other things, but in his efforts to do so he had found that the heavens, the first of God's works, were laid out more beautifully than other things. Presumably this meant that the grand design of the heavens was more suited for understanding by the human mind than the design in the small details of more common things. Cf. Book IV, note 71.

For the kinds of harmonies had to be distinguished, by Axiom XX. Yet that could not have been done except by the sixths, by XXIII. Therefore, since one of them, 5:8, has been taken by the Earth and Venus, the closest planets, which are icosahedric by XV, the other, 3:5, should also have been attributed to them. But not between the converging and diverging extreme motions, but between the extreme motions on the same side, one between the motions at aphelion, the other between those at perihelion, by XXIV. In addition the harmony 3:5 is also akin to the icosahedron, inasmuch as both are of the pentagonic group. See Chapter II.

Here is the reason why precise harmonies are rather found between the motions of these two at aphelion and perihelion, but not between their converging motions, as in the superior planets.[143]

XXVIII. Proposition

For the Earth the motions' own proportion agreed with 14:15, about; for Venus about 35:36.

For these two ought to have distinguished the kinds of harmonies, by what has already been stated. Then by XXVI the Earth in fact as the superior planet ought to have got the interval 2916:3125, that is nearly 14:15; whereas Venus, as the inferior, ought to have got the interval 243:250, that is 35:36, very nearly.[144]

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[143] As the motions of the Earth and Venus are of fundamental importance for the building of the hard and soft scales, it is worthwhile to reflect on the reasoning by which Kepler has demonstrated their motions *a priori*. In order to build the hard and soft scales, two harmonies are required that differ by a diesis. The only harmonies having this property are the thirds and the sixths. Since no pair of planets can have the major and minor thirds as harmonies (VI), it follows that there must be a pair of planets having the major and minor sixths as harmonies (XXIII). Next he shows that this pair of planets must form the major sixth from their aphelion motions and the minor sixth from their perihelion motions. For if the consonances were formed from the divergent and convergent motions, each planet would vary its own motion corresponding to half a diesis, which would not be melodic (XXIV).

Turning now to the proportions of the motions of the individual planets, he shows that these must be smaller than a minor tone and a semitone respectively (XXV), otherwise the convergent and divergent motions would also produce consonances, which has already been excluded (XXIII). Now there are only two melodic intervals smaller than the minor tone and the semitone. These are the semitone and the diesis. However, the difference must be exactly a diesis in order to build the hard and soft scales. This is achieved by substituting a double diesis for the semitone. But this is too large. So Kepler subtracts a comma from both the double diesis and the diesis. The difference remains exactly a diesis, but the *double diesis—comma* is too wide by a comma and the *diesis—comma* is too narrow by a comma. A difference of a comma from the required interval is, of course, musically acceptable. The proportions of the motions of the two planets are thus found to be 2916:3125 and 243:250 (XXVI). Finally, Kepler shows that the Earth and Venus are the only pair that can produce the major and minor sixths (XXVII), so that the proportions 2916:3125 and 243:250 are those of the motions of the Earth and Venus, respectively.

[144] The Earth and Venus are the only pair of planets in which the aphelion motions and the perihelion motions, rather than the convergent and divergent motions,

Here is the reason why these two planets have such small eccentricities, and resulting from them, small distances or proportions as their extreme motions' own, while nevertheless Mars which is the next superior to the Earth, and Mercury which is the next inferior to Venus, have ones which are outstanding and greatest of all. However, astronomy confirms that this is true; for in Chapter IV the Earth clearly had 14:15, and Venus 34:35, which astronomical accuracy will scarcely be able to distinguish from 35:36.[145]

XXIX. Proposition

The greater harmony of the motions of Mars and the Earth, that is of their diverging motions, could not have been among those greater than 5:12.

By Proposition XVII above it was not one of the lesser proportions: but now it is not one of those greater, either. For the other common proportion of these planets, or the lesser, 2:3, multiplied by Mars' own proportion, which by XIV exceeds 18:25, makes more than 12:25, that is, 60:125. Therefore, multiply it by the Earth's own proportion, 14:15, that is 56:60, by what has already been stated: the product is more than 56:125. That is very nearly 4:9, that is to say more than an octave and a tone, by a little. But the harmony next greater than an octave and a tone is 5:12, a diapason with a soft third.

Note that I do not say that this proportion is neither greater nor lesser than 5:12; but what I do say is that if it must necessarily be harmonic, no other harmony would agree with it.

XXX. Proposition

Mercury's motions' own proportion ought to have been greater than all the others' own proportions.

For by XVI Venus' and Mercury's own proportions combined ought to have made about 5:12. But Venus' own separately is only 243:250, that is 1458:1500; and that divided into 5:12, that is into 625:1500, leaves 625:1458, which is greater than a diapason with a major tone, for Mercury alone, whereas Mars's own proportion, which among the other planets is the greatest of all, is less than the sesquialterate proportion 2:3, that is a diapente.

And in fact for Venus and Mercury, the lowest planets, their own proportions combined equal the four highest planets' own proportions combined, nearly. For as will now immediately be apparent, Saturn and Jupiter's own proportions combined exceed 2:3. Mars's own proportion falls considerably short of 2:3: the product is 4:9, that is 60:135. Multiply by that of the Earth, 14:15, that is 56:60: the product is 56:135, which is a little more than 5:12, and as we have

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form harmonies. The proportion of the aphelion motions is 0.602 (major sixth = 0.600), and the proportion of the perihelion motions is 0.628 (minor sixth = 0.625).

[145] For the Earth, observation gives 0.931 and theory gives 0.933. For Venus, observation gives 0.971 and theory gives 0.972.

just seen is the product of Venus and Mercury's own proportions. However, that was not sought, nor taken from some separate and special archetype of beauty, but emerges spontaneously, by the necessity of causes connected with the harmonies confirmed so far.[146]

XXXI. Proposition

The motion of the Earth at aphelion ought to have been in harmony with Saturn's at aphelion over several diapasons.

For there must have been universal harmonies, by XVIII, and hence also harmony of Saturn with the Earth and Venus. But if one of the extreme motions of Saturn had been in harmony with neither of the latter, that would have been less harmonic than if both of its extreme motions were in harmony with those planets, by Axiom I. Then Saturn should have been in harmony at both its extremes: in its motion at aphelion with one of those two planets, at perihelion with the remaining one, since there was no impediment, inasmuch as it was the motion of the first planet. Therefore, these harmonies will be either identical in sound or different in sound, that is in either the proportion of continuous doubling or another proportion. But both cannot be in another proportion, for between the terms of 3:5 (defining the greater harmony between the motions of the Earth and Venus at aphelion, by XXVII) there cannot exist two harmonic means, as a sixth cannot be divided into three harmonic intervals. See Book III. Therefore, Saturn could not have made a diapason with harmonic means between 3 and 5 with both its motions; but for its motions to be in harmony both with the 3 of the Earth and with the 5 of Venus, one of the motions must be in identical harmony with one of the terms, that is with one of the planets mentioned themselves, or in harmony over several diapasons. Now since identical harmonies are more outstanding, they will also have to be established between the more outstanding extremes of the motions, that is between the motions at aphelion, both because they hold the position of preeminence on account of the loftiness of the planets, and because they claim the harmony 3:5, with which we are now dealing as the greater harmony of the Earth and Venus, as their own, in a sense, and as their privilege. For although that harmony also agrees with the motion of Venus at perihelion and some intermediate motion of the Earth, by XXVII, yet the beginning is from the extremes of the motions, and the intermediate motions give precedence to the chief ones. In that case since on the one hand we have the motion at aphelion of Saturn the highest planet, on the other hand it is the motion of the Earth at aphelion rather than that of Venus which must be coupled with it, because of these two, which distinguish their kind of harmony, the former is again the higher. There is also another more immediate reason: that the a posteriori *arguments, in which we are now engaged, do indeed modify the* a priori *arguments, but only in the least im-*

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[146] Here is a rare occasion on which Kepler is willing to consider a harmonic relationship to be accidental. For though it is demonstrated from the archetype, the archetype was not designed to produce it.

portant points, because it is a question of harmony, in respect of intervals which are less than all the melodic intervals. But by the a priori *arguments the motion at aphelion, not of Venus, but of the Earth, was close to the harmony of several diapasons which had to be established with the motion of Saturn at aphelion. For multiply together into a single product first the Saturnine motions' own proportion 4:5, that is from the motion of Saturn at aphelion to its motion at perihelion, by XI; second, the proportion of the converging motions of Saturn and Jupiter, 1:2, that is from the motion at perihelion of Saturn to that of Jupiter at aphelion, by VIII; third, the proportion of the diverging motions of Jupiter and Mars, 1:8, that is from the motion at aphelion of Jupiter to the motion at perihelion of Mars, by XIII; and fourth, the proportion of the converging motions of Mars and the Earth, 2:3, that is from the motion at perihelion of Mars to the motion at aphelion of the Earth, by XV. You will find the total product between the motion at aphelion of Saturn and the motion at aphelion of the Earth is the proportion 1:30, which falls short by not more than 30:32, that is 15:16, or a semitone, of being 1:32, or five diapasons. Then if a semitone, divided into parts smaller than the least melodic interval, were to be added to these four elements, there will be between the motions in the proposition of Saturn and the Earth at aphelion a perfect harmony of a fivefold diapason. But for the same motion at aphelion of Saturn to make several diapasons with the motion at aphelion of Venus, it would have been necessary by the* a priori *arguments to tear away almost a complete diatessaron. For if 3:5, which is between the motions at aphelion of the Earth and Venus, is multiplied by the total 1:30 produced by combining the four previous elements, the result, as if from the* a priori *arguments, is 1:50 between the motions at aphelion of Saturn and Venus, an interval which differs from 1:32, a fivefold diapason, by 32:50, that is by 16:25, which is a diapente with a diesis, and differs from a sixfold diapason, or 1:64, by 50:64, which is 25:32, or a diatessaron minus one diesis. Therefore, the identical harmony had to be set up not between the motions at aphelion of Venus and Saturn, but between those of the Earth and Saturn, so that there would remain for Saturn a harmony different in sound with Venus.*

XXXII. Proposition

In the universal harmonies of the planets of the soft kind, the motion of Saturn absolutely at aphelion could not have been exactly in harmony with the other planets.

For the Earth in its motion at aphelion does not coincide with universal harmony of the soft kind, because the motions of the Earth and Venus at aphelion make the interval 3:5, of the hard kind, by XXVII. However, Saturn in its motion at aphelion makes identical harmony with the motion of the Earth at aphelion, by XXXI. Then neither does Saturn coincide in its motion at aphelion. However, there succeeds in place of the motion at aphelion a more vigorous motion of Saturn, very close to that at aphelion, and also to the soft kind, as became apparent in Chapter VII.

XXXIII. Proposition

The hard kind of harmonies and of musical scale is closely related to motions at aphelion, the soft kind to those at perihelion.[117]

For although a hard harmony is established not only between the motion of the Earth at aphelion and that of Venus at aphelion, but also between the motions of the Earth lower than aphelion and those of Venus lower than aphelion, right down to its perihelion; and on the other hand a soft harmony not only between the motion of Venus at perihelion and that of the Earth at perihelion, but also between the higher motions of Venus right up to the aphelion and the higher motions of the Earth, by XXVII; yet the proper and obvious representation of kind belongs only to the extreme motions of each planet, by XX and XXIV. Therefore, the proper representation of the hard kind belongs only to the motions at aphelion, and the proper representation of the soft kind only to the motions at perihelion.

XXXIV. Proposition

The hard kind is more closely related to the superior planet in a comparison between two, and the soft kind to the inferior.

For because the hard kind properly belongs to the motions at aphelion, and the soft kind to those at perihelion, by what has already been stated, and those at aphelion are slower and more deliberate than those at perihelion, therefore the hard kind belongs to the slower motions, the soft to the quicker. But the superior of the two planets is more closely related to the slow motions, and the inferior to the quicker, because always in the world height is accompanied by slowness of the planet's own motion. Then of two which fit both kinds that which is superior is more closely related to the hard kind of scale, and that which is inferior to the soft. Further, the hard kind uses greater intervals, 4:5 and 3:5, the soft lesser intervals, 5:6 and 5:8. But in addition the superior planet also has a greater sphere and slower, that is greater, motions, and a more extended orbit; and those to which great things are appropriate in each case join in a closer relationship between themselves.

XXXV. Proposition

Saturn along with the Earth embraces the hard kind in a closer relationship, Jupiter with Venus the soft kind.

For first, the Earth in comparison with Venus, and representing along with Venus both kinds, is the superior. Therefore, the Earth chiefly embraces the hard kind, and Venus the soft kind, by what has already been stated. Now Saturn in its motion at aphelion is consonant over a diapason with the Earth's motion at aphelion, by XXXI. Hence by XXXIII Saturn also embraces the hard kind. Second, Saturn in its motion at aphelion, by the same Proposition, cherishes

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[117] This and the previous proposition are needed to justify beginning the soft scale on the note represented by the perihelion motion of Saturn, as Kepler found to be indicated by empirical considerations in Chapter V.

the hard kind more, and rejects the soft kind, by XXXII. Therefore, it is more closely related to the hard kind than to the soft kind, because the kinds are properly represented by the extreme motions.

In that case, as far as Jupiter is concerned, in comparison with Saturn it is inferior. Then as the hard kind ought to belong to Saturn, so the soft ought to belong to Jupiter, by what has already been stated.

XXXVI. Proposition

The motion of Jupiter at perihelion ought to have agreed with that of Venus at perihelion in a single musical scale, but not in the same harmony as well; and much less so with that of the Earth at perihelion.

For as Jupiter ought to have belonged chiefly to the soft kind, by what has been stated previously, and the motions at perihelion are closely related to that kind, by XXXIII, therefore Jupiter by its motion at perihelion ought to have represented the scale of the soft kind, that is to say a definite position or sound in it. But the motions at perihelion of Venus and the Earth as well represent the same scale, by XXVII. Then the motion of Jupiter at perihelion had to be associated with the motions of these latter planets at perihelion in the same tuning. However, it could not have set up a harmony with the motion of Venus at perihelion. For as by VIII it ought to have made about 1:3 with the motion of Saturn at aphelion, that is, the note d *of the system in which the motion of Saturn at aphelion made the note* G, *but the motion of Venus at aphelion the note* e, *therefore it came close to the note* e *within the interval of the smallest harmony. For that is 5:6; but the interval between* d *and* e *is much less, that is to say 9:10, a tone. And although in the tuning at perihelion Venus is raised above its* e *in the tuning at aphelion, yet this rise is less than a diesis, by XXVIII. However, a diesis (and something less than that) combined with a minor tone does not yet equal the interval of the smallest harmony, 5:6. Therefore, the motion at perihelion of Jupiter could not have protected the position with the motion of Saturn at aphelion except by means of 1:3 while still being in harmony with Venus. But neither could it with the Earth. For if the motion at perihelion of Jupiter has been fitted to the scale of the motion at perihelion of Venus, in the same tuning, in such a way that within less than the amount of the smallest interval it protects its interval with the motion of Saturn at aphelion, 1:3, that is to say separated from the motion of Venus at perihelion by a minor tone, that is 9:10 or 36:40 (in addition to some diapasons) on the lower side, the motion of the Earth at perihelion is of course separated from that same motion of Venus at perihelion by 5:8, that is by 25:40. Thus the motions at perihelion of the Earth and Jupiter will be separated by 25:36, in addition to several diapasons. However, that is not harmonic, as it is double 5:6, or a diapente, diminished by one diesis.*

XXXVII. Proposition

The sum of Saturn's and Jupiter's own harmonies, 2:3, and their greater common harmony, 1:3, should have been increased by an interval equal to the interval of Venus.

For Venus by its motion at aphelion properly assists the representation of the hard kind, at perihelion of the soft kind, by XXVII and XXXIII. But Saturn by its motion at aphelion ought also to have agreed with the hard kind, and thus with the motion at aphelion of Venus, by XXXV; but Jupiter by its motion at perihelion with the motion at perihelion of Venus, by what has already been stated. Therefore, the factor of the interval which Venus makes between its aphelion and perihelion is also the factor by which it is necessary to increase the motion of Jupiter mentioned, which combined with the motion of Saturn at aphelion makes 1:3 with the actual motion of Jupiter at perihelion. But the harmony of the converging motions of Jupiter and Saturn is precisely 1:2 by VIII. Therefore, subtraction of the interval 1:2 from that, which is more than 1:3, leaves a remainder which is more than 2:3 by the sum of the intervals of each planet's own proportions.

Above, in Proposition XXVIII, Venus' motions' own proportion was 243:250, or very nearly 35:36. However, in Chapter IV between the motion of Saturn at aphelion and that of Jupiter at perihelion was found an excess over 1:3 which was a little greater, that is to say between 26:27 and 27:28. But if a single second—and I don't know whether astronomy can detect it—is added to the motion of Saturn at aphelion, the quantity here prescribed is plainly equal to it.

XXXVIII. Proposition

The surplus factor of 243:250 in the product of Saturn's and Jupiter's own motions, which up to this point was established from first principles as 2:3, had to be distributed among the planets in the following way: a comma, 80:81, from it was given to Saturn, and to Jupiter the quotient, 19683:20000, or 62:63 nearly.

That this factor had to be distributed between both planets follows from XIX, so that both could coincide within a certain range with the universal harmonies of the kind related to it. But the interval 243:250 is less than all the melodic intervals. Therefore, no harmonic laws are left by which to divide it into two melodic parts, with the sole exception of those which were needed above in Proposition XXVI for the division of a diesis, 24:25, that is to say that it should change into a comma, 80:81 (which is one, and indeed the chief, of those which are used for melodic intervals[148]) and a quotient of 19683:2000, which is a little more than a comma, that is 62:63 nearly. However, not two commas but one comma had to be split off, so that the parts should not become too unequal, since Saturn's and Jupiter's own proportions are very nearly equal, according to Axiom X extended also to melodic intervals and parts tinier than they, and also at the same time because a comma is defined by the intervals of a major tone and a minor tone, but not so two commas. Furthermore, to Saturn

[148] For the division of intervals smaller than the melodic intervals, Kepler uses the comma rather than an arbitrary division because, as he has claimed in XXVI, even if the nature of harmony has not provided weightier arguments for the division of such intervals, God has ordered nothing without a reason.

as the higher and more powerful planet ought to have belonged for preference not the greater of these parts, although it did have as its own 4:5 which is the greater, but the prior and more beautiful, that is more harmonic. For in Axiom X, consideration of priority and harmonic perfection takes precedence; consideration of size comes last, because there is no beauty in size by itself. Thus the motions of Saturn become 64:81, an impure major third, as we have called it in Book III, Chapter XII; but those of Jupiter 6561:8000.

I do not know whether it should be mentioned among the reasons for the addition of a comma to Saturn that it was to enable the extreme distances of Saturn to set up the proportion 8:9, a major tone; or rather it came about spontaneously from the antecedent causes of the motions. You therefore have here in place of a corollary rather a reason why above in Chapter IV, page 420, the intervals of Saturn were found to embrace the proportion of a major tone, very nearly.

XXXIX. Proposition

In the universal harmonies of the planets, of the hard kind, Saturn could not be in harmony in its motion exactly at perihelion, nor Jupiter in its motion exactly at aphelion.[119]

For since the motion of Saturn at aphelion should have been exactly in harmony with the motions of the Earth and Venus at aphelion, by XXXI, the motion of Saturn which is more hurried than its motion at aphelion by one hard third, 4:5, will also be in harmony with those same planets; for the motions of the Earth and Venus at aphelion make a hard sixth, which by what has been shown in Book III is divisible into a diatessaron and a hard third. Then the motion of Saturn, which until this point is quicker than the motion which has now been harmonized, though by less than the amount of a melodic interval, will not be exactly in harmony. But the actual motion of Saturn at perihelion is such, because it is separated from its motion at aphelion by more than the interval 4:5, that is to say more than one comma, 80:81 (which is less than the smallest melodic interval) by XXXVIII. Therefore, the motion of Saturn exactly at perihelion is not in harmony. But neither is the motion of Jupiter exactly at aphelion; for it is consonant over a perfect diapason, by VIII, with the motion at perihelion of Saturn, which is not exactly consonant. Hence by what has been said in Book III it cannot itself be exactly consonant either.

XL. Proposition

To the common harmony of the diverging motions of Jupiter and Mars, 1:8, a triple diapason, confirmed by the *a priori* arguments, a Platonic limma had to be added.

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[119] In the universal harmonies of the hard kind, established from the data (Chapter VII), Kepler had found that the perihelion motion of Saturn and the aphelion motion of Jupiter fitted in the intermediate tuning.

For between the motions at aphelion of Saturn and the Earth there had to be 1:32, that is 12:384, by XXXI; but from the motion of the Earth at aphelion to the motion of Mars at perihelion there had to be 3:2, that is 384:256, by XV; and from the motion at aphelion of Saturn to its motion at perihelion 4:5 or 12:15, together with the extra factor, by XXXVIII; and last, from the motion at perihelion of Saturn to the motion at aphelion of Jupiter 1:2, or 15:30, by VIII. Therefore, the quotient, from the motion at aphelion of Jupiter to the motion at perihelion of Mars is 30:256, after dividing it by Saturn's extra factor. But 30:256 exceeds 32:256, that is 1:8, by the factor 30:32, that is 15:16 or 240:256, which is a semitone. Therefore, division of 240:256 by Saturn's extra factor, which by Proposition XXXVIII should have been 80:81, that is 240:243, leaves 243:256. But that is a Platonic limma, that is 19:20, nearly: see Book III. Therefore, a Platonic limma had to be added to the 1:8.

Thus the greater proportion of Jupiter and Mars, that is the proportion of their diverging motions, ought to be 243:2048, which is in a way a mean between 243:2187 and 243:1944 that is between 1:9 and 1:8. Of these the former was required above by direct proportion,[150] the latter by harmonic melodicity, which is closer to hand.

XLI. Proposition

Mars's motions' own proportion was necessarily made the square of the harmonic proportion 5:6, that is to say 25:36.

For because the proportion of the diverging motions of Jupiter and Mars had to be 243:2048, that is 729:6144, by the previous proposition, and that of their converging motions 5:24, that is 1280:6144, by XIII, therefore the product of their own proportions was necessarily 729:1280, or 72900:128000. But Jupiter's own proportion alone had to be 6561:8000, that is 104976:128000, by XXXVIII. Then if this proportion of Jupiter's is divided into the product of both, the quotient is Mars's own proportion, 72900:104796, that is 25:36, the square root of which is 5:6.

Alternatively as follows. From the motion of Saturn at aphelion to that of the Earth at aphelion is 1:32 or 120:3840. From the same motion of Saturn to the motion at perihelion of Jupiter is 1:3, or 120:360, with its excess factor surplus. Now from that to the motion at aphelion of Mars is 5:24, or 360:1728. Therefore, from the motion of Mars at aphelion to the motion of the Earth at aphelion is the quotient, 1728:3840, divided by the extra factor in the proportion of the diverging motions of Saturn and Jupiter. But from the same motion

[150] See Proposition XIII, where Kepler remarked that it was appropriate to fit proportions of motions to the tetrahedron that were the squares of the proportions of motions fitted to the cube. In the case of the cube, interpolated between Saturn and Jupiter, the proportions are 1:2 and 1:3. The squares of these, 1:4 and 1:9, according to Kepler's analogy should be the proportions fitted to the tetrahedron, interpolated between Jupiter and Mars. But 1:9 is not harmonic and 1:4 has been taken for the octahedron. Neighboring harmonies must therefore be taken. The neighbors of 1:9 are 1:8 and 1:10. Kepler gives reasons for choosing 1:8.

at aphelion of the Earth to the motion at perihelion of Mars is 3:2, that is 3840:2560. Then between the motions of Mars at aphelion and perihelion the quotient will be the proportion 1728:2560, that is 27:40, or 81:120, divided by the extra factor mentioned. But 81:120 is a comma less than 80:120, or 2:3. Then if a comma were divided into 2:3, and the excess factor mentioned (which by XXXVIII is equal to Venus' own proportion) were also divided into it, the quotient is Mars's own proportion. But Venus' own proportion is a diesis diminished by a comma, by XXVI. Now a comma and a diesis diminished by a comma make a whole diesis, 24:25. Then if you divide 2:3, that is 24:36, by a diesis, 24:25, the quotient will be Mars's own proportion, 25:36 as before; and the root of that, 5:6, is allotted to the intervals, by Chapter III.[151]

Look, here is another reason why above, in Chapter IV, page 424, the extreme distances of Mars were discovered to embrace the harmonic proportion 5:6.[152]

XLII. Proposition

The greater common proportion of Mars and the Earth, or that of their diverging motions, was necessarily made 54:125, less than the harmony 5:12 confirmed by the *a priori* arguments.

For Mars's own proportion had to be a diapente, from which a diesis was removed, by the previous proposition. However, the common proportion of the converging motions of Mars and the Earth, or the lesser common proportion, had to be a diapente, 2:3, by XV. Last, the Earth's own proportion is a doubled diesis, from which a comma has been removed, by XXVI and XXVIII. Now of these elements is composed the greater proportion, or that of the diverging motions, of Mars and the Earth; and it comes to two diapentes (or 4:9, that is 108:243) together with one diesis which is mutilated of a comma, that is together with 243:250. That is, it comes to 108:250, or 54:125, that is 608:1500. But that is less than 625:1500, that is, than 5:12, by the factor of 608:625: and that is nearly 36:37, less than the smallest melodic interval.

XLIII. Proposition

The motion of Mars at aphelion could not agree with a universal harmony; yet it was necessary for it to be in accord to a certain extent with the scale of the soft kind.

For because the motion at perihelion of Jupiter holds the position of d *in*

[151] According to Number Six (2) of Chapter III, the proportion of the apparent daily arcs on a given eccentric is almost exactly the square of the inverse proportion of their distances from the sun.

[152] In this proposition, Kepler has shown that the proportion of the aphelion and perihelion motions of Mars does not in fact represent the harmonic interval of a diapente originally allotted to it. For a diapente diminished by a diesis is not harmonic. But this is compensated for by the fact that the perihelion and aphelion distances of Mars represent a harmonic interval, namely the minor third. Cf. note 74.

the high tuning in the soft kind, and in fact between it and the motion of Mars at aphelion there had to be the harmony 5:24, then the motion of Mars at aphelion holds the position of the impure f *in the same high tuning. I say, impure; for in Book III, Chapter XII, when the impure consonances were enumerated, and removed from the composition of the systems, some were omitted which do exist in the actual simple natural system. Thus the reader should write in, after the line which finishes thus, "81:120," the following*: if you divide this by 4:5 or 32:40, the quotient is 27:32, a narrow soft third, which is between *d* and *f* or *c*♮ and *e* or *a* and *c* even in the simple octave.[153]

And in the table below that the following should occupy the first line:

For 5:6 there is 27:32, undersize.

From which it is evident that in the natural system the genuine note f, *as it is defined in accordance with my basic principles, constitutes with the note* d *an undersize or impure soft third. Therefore, since between the motion at perihelion of Jupiter, set up on the genuine note* d, *and the motion at aphelion of Mars, there is a perfect soft third above the double diapason, and not an undersize one, by XIII, it follows that Mars by its motion at aphelion signifies the position which is one comma higher than the genuine note* f. *Thus it will hold nothing but the impure* f; *and so it is in accord with this scale, not directly but at least to a certain extent. However, it does not enter a universal harmony, either pure or impure. For the motion of Venus at perihelion holds the position* e *in this tuning. But there is a dissonance between* e *and* f, *as they are neighbors. Therefore, Mars is in dissonance with the motion of one of the planets, that is with the motion of Venus at perihelion. But it is also in dissonance with the other motions of Venus, for they lag by one comma less than one diesis. Hence, since between the motion of Venus at perihelion and that of Mars at aphelion there is a semitone and a comma, therefore between the motion of Venus at aphelion and the motion of Mars at aphelion there will be a semitone and a diesis (disregarding the octaves), that is a minor tone, which is still a dissonant interval. Now the motion of Mars at aphelion is in accord with the scale of the soft kind to that extent, but not with that of the hard as well. For since the motion at aphelion of Venus agrees with* e *of the hard kind, whereas the motion of Mars at aphelion (disregarding the octaves) has been made higher than* e *by a minor tone, therefore the motion of Mars at aphelion in this tuning would necessarily fall as a mean between* f *and* f♮, *making with* g *(which in this tuning is taken by the motion of the Earth at aphelion) the interval 25:27, which is plainly unmelodic, that is a major tone from which a diesis has been subtracted.*

In the same way it will be proved that the motion of Mars at aphelion is also at odds with the motions of the Earth. For because with the motion of Venus at perihelion it makes a semitone and a comma, by what has been said, that is 14:15, but the motions of the Earth and Venus at perihelion make a soft sixth, 5:8 or 15:24, by XXVII, therefore the motion of Mars at aphelion with the motion of the Earth at perihelion (with octaves added to the former) will make 14:24, or 7:12, an unmelodic interval, still less harmonic, as is 7:6 also.

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[153] The soft third is narrow by a comma.

For anything between 5:6 and 8:9, as 6:7 is in this instance, is dissonant and unmelodic. But neither can any other motion of the Earth be in harmony with the motion at aphelion of Mars. For it has been stated above that it makes with the motion of the Earth at aphelion 25:27, which is unmelodic (disregarding the octaves): but in this case from 6:7 or 24:28 up to 25:27 all intervals are less than the smallest harmonic interval.

XLIV. Corollary

It is therefore clear from this proposition XLIII, on Jupiter and Mars, and from XXXIX on Saturn and Jupiter, and from XXXVI on Jupiter and the Earth, and from XXXII on Saturn, why above in Chapter V it was discovered that neither did all the extreme motions of the planets fit perfectly a single natural system or musical scale, nor did all those which fitted a system in the same tuning divide up the positions in a natural pattern, or produce a purely natural succession of melodic intervals. For the reasons why individual planets acquired individual harmonies, why also all the planets acquired universal harmonies, and last, why the universal harmonies also acquired two kinds, hard and soft, are prior; and these being granted, now any kind of accommodation to a single natural system is prevented. But if those reasons had not necessarily taken precedence, there is no doubt that a single system, and a single tuning to it, would have embraced the extreme motions of all the planets; or if two systems were needed, for the two kinds of melody, hard and soft, the actual order of the natural scale would have been expressed not only in the one scale of the hard kind but also in the other scale of the soft kind. Therefore, you have here the reasons, promised in the said Chapter V, for the disagreements over very small intervals, smaller in fact than all the melodic intervals.[154]

XLV. Proposition

The greater common proportion of Venus and Mercury, a double diapason, and also Mercury's own proportion, by Propositions XII and XVI confirmed by *a priori* arguments above, had to be multiplied by an interval equal to the interval of Venus, in such a way that Mercury's own proportion became a perfect 5:12, and thus Mercury was in harmony in both its motions with the motion of Venus at perihelion alone.

[154] That is, smaller than a diesis. Clearly, the accommodation of all the motions to a single natural scale was an ideal that even God could not attain without sacrificing the more important arrangements that gave harmonies to the individual planets and universal harmonies of the two kinds, hard and soft. For in the construction of the geometrical archetype, the laws of logic and mathematics imposed conditions that were part of that necessity against which, in the words of the old poetic expression, "not even the gods make war." See Plato, *Protagoras*, 345D.

For because the motion of Saturn at aphelion ought to have been in harmony with the motion of the Earth at aphelion, that of the outermost planet, which is circumscribed about its figure and highest, with the highest motion of the Earth, which distinguishes the classes of figures, it follows by the laws of opposites that the motion of Mercury at perihelion agrees with the motion of the Earth at perihelion, that is the motion of the inmost planet, inscribed in its figure, lowest, and closest to the Sun, with the lowest motion of the Earth, the common boundary, the former indeed to mark out the hard kind of harmonies, the latter the soft kind, by Propositions XXXIII and XXXIV. But the motion of Venus at perihelion ought to have been consonant with the motion of the Earth at perihelion in the harmony 5:8, by XXVII. Then the motion of Mercury at perihelion also ought to have been combined with the motion at perihelion of Venus in a single scale. However, by Proposition XII from the a priori *arguments the harmony of the diverging motions of Venus and Mercury was specified as 1:4. Then by these* a posteriori *arguments that had in this case to be leavened by the addition of the whole of the interval of Venus. Therefore, there is no longer a perfect disdiapason from the motion at aphelion of Venus, but from its motion at perihelion, to the motion at perihelion of Mercury. But the harmony of the converging motions, 3:5, is also perfect, by Proposition XV. Therefore, on dividing that into 1:4, the quotient is Mercury's own harmony alone, 5:12, which is also perfect, but no longer (as by Proposition XVI through* a priori *arguments) diminished by Venus' own proportion.*

Another argument. Just as Saturn and Jupiter alone are not touched at all on the outside by the dodecahedron and icosahedron, which are a married couple, so Mercury alone is not touched by the same figures inside. For they touch Mars, the Earth, and Venus, the first inside, the last outside, the middle one on both sides. Therefore, just as Saturn's and Jupiter's motions' own proportions, which were supported by the cube and tetrahedron, were increased by something equal to Venus' own proportion in corresponding shares, so in this case the solitary Mercury's own proportion, which is contained within the octahedron, a figure allied to the cube and tetrahedron, ought to have been increased by the same factor. That is, just as the octahedron, a single figure among the secondaries, sustains the role of two among the primaries, the cube and tetrahedron—on which see Chapter I—so also among the inferior planets Mercury alone takes the place of two of the superior planets, that is to say Saturn and Jupiter.

Third, just as Saturn, the highest planet, ought to have been in harmony in its motion at aphelion over several diapasons, that is, in the proportion 1:32, by continuous doubling, with the motion, also at aphelion, of the higher, and closer to itself, of the two which change the kind of harmony, by XXXI; so the other way round, Mercury the lowest planet ought to have been in harmony in its motion at perihelion, again over several diapasons, that is in the proportion 1:4, also by continuous doubling, with the motion at perihelion of the lower, and similarly closer to itself, of the two which change the harmony.

Fourth, only the individual extreme motions of the three superior planets, Saturn, Jupiter and Mars, agree in universal harmonies; therefore both the extremes of the lower and lone planet, that is Mercury, ought to have agreed in the same, for those in between, the Earth and Venus, ought to have changed the kind of harmonies, by XXXIII and XXXIV.

Last, in the three pairs of the superior planets perfect harmonies were found among the converging motions, but leavened harmonies among the diverging motions, also the individual planets' own proportions; therefore in the two pairs of the inferior planets, the other way round, perfect harmonies ought to have been found not chiefly between the converging motions, nor between the diverging motions, but between motions on the same side.[155] *And because two perfect harmonies ought to have belonged to the Earth and Venus, hence Venus and Mercury also ought to have had two perfect harmonies. And the former two indeed ought to have been allotted a perfect harmony both between their motions at aphelion and between those at perihelion, because they ought to have changed the kind of harmony; whereas Venus and Mercury, as they do not change the kind of harmony, did not also require perfect harmonies between both pairs, both of the motions at aphelion and of those at perihelion. But instead of a perfect harmony of the motions at aphelion, inasmuch as it had already been leavened, there succeeded a perfect harmony of the converging motions. So, just as Venus, the superior among the inferior planets, has as its motions' own proportion the smallest of all, by XXVIII, but Mercury, the inferior of the inferior planets, has been allotted as its own proportion the greatest of all, by XXX; so also Venus' own proportion was of all the planets' own motions the most imperfect, or the most remote from harmonies, but Mercury's own proportion was of all the planets' own proportions the most perfect, that is absolute harmony, without leaven; and so in the end the patterns were opposite on all sides.*

For thus has He who is before ages and to all ages has embellished the mighty works of His wisdom: nothing is redundant, nothing is deficient, and there is no place for any criticism. How desirable are His works, and so forth, all balanced one against another, and none lacks its opposite;[156] of every one He has established (*He has confirmed with the best arguments*) the goodness (*their furnishing and comeliness*) and who shall be sated with seeing their glory?

XLVI. Axiom

The placing of the solid figures among the planetary spheres, if it is unrestricted, and not prevented by the necessities of preceding causes, ought in perfection to follow the analogy of the geometrical inscriptions and circumscriptions, and therefore the terms of the proportion of the inscribed spheres to the circumscribed spheres.[157]

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[155] That is, the perihelion motions or the aphelion motions.

[156] Kepler here stresses the symmetry of the musical harmony in relation to the Earth. As befits the abode of man, the Earth occupies a privileged position in respect of the musical harmony as well as in relation to the solids, standing between the primaries and the secondaries.

[157] This axiom asserts the role of the regular polyhedra in the determination of the planetary distances and enables Kepler to explain the discrepancies between the distances that would arise from a strict interpolation and the distances revealed by observations as necessary consequences of the harmonic proportions that were given priority in the construction of the world.

For nothing is more fitting than that the physical inscription exactly represents the geometrical, as a printed work does its type.

XLVII. Proposition

If the inscription of the figures among the planets was unrestricted, the tetrahedron ought to have touched the sphere of the perihelion of Jupiter above exactly at its vertices, and below the sphere of the aphelion of Mars exactly at the centers of its faces. However, the cube and the octahedron, resting at their vertices each on the sphere of the perihelion of its own planet, ought to have penetrated at the centers of their faces the sphere of its interior planet, in such a way that those centers are situated between the spheres of its aphelion and perihelion. On the other hand, the dodecahedron and icosahedron, which at their vertices make contact with the spheres of perihelion of their planets on the outside, clearly ought not to have touched with the centers of their faces the spheres of aphelion of their interior planets. Last, the dodecahedric hedgehog, which stands with its vertices on the sphere of perihelion of Mars, ought to have come very close to the sphere of aphelion of Venus with the midpoints of its inverted edges,[158] which separate the two spikes in each case.

For the tetrahedron is the middle one of the primary figures, both by its origin and by its position in the world. It ought therefore, if there was no impediment, to have moved apart both the regions of Jupiter and Mars equally. As the cube was upwards and further out with respect to the latter, the dodecahedron downwards and further in, then it was proper for inscription within them to bring about opposite effects, between which the tetrahedron held the mean, and for one of the figures to go beyond the inscription and the other to fall short of it, that is, for one to penetrate the interior sphere to a certain extent, and the other not to reach it. And because the octahedron is akin to the cube, having an equal proportion between its spheres, but the icosahedron to the dodecahedron, therefore if the cube has any element of perfection in its inscription, the octahedron ought to have had the same; and if the dodecahedron has any, so ought the icosahedron. The position of the octahedron is also very similar to the position of the cube, and that of the icosahedron to the position of the dodecahedron, because as the cube holds one boundary towards the outside, so the octahedron holds the other extreme towards the inside parts of the world, whereas the dodecahedron and the icosahedron come between. It is therefore appropriate that their mode of inscription should be similar, in the former case penetrating the interior of the planetary sphere, in the latter falling short of it. However, the hedgehog, which with the tips of its spikes represents an icosahedron, with their bases a dodecahedron, ought to have filled, embraced, or arranged both regions also, that between Mars and the Earth, which are

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[158] By the inverted edges is meant the edges of the dodecahedron forming the nucleus.

attributed to the dodecahedron, and also that between the Earth and Venus, which are attributed to the icosahedron. However, which of the opposites is appropriate to which alliance, the previous axiom makes clear. For the tetrahedron, having an expressible inscribed sphere, has been allotted the place in the middle among the primaries, attended on both sides by the figures of the incommensurable spheres, of which the outer is the cube, the interior the dodecahedron, by Chapter 1 of this Book. Now this geometrical property, the expressibility of the inscribed sphere, represents in Nature the perfect inscription of the planetary sphere. Therefore, the cube and its allied figure have inscribed spheres which are expressible only as roots, that is only in the square. Therefore, they ought to represent semiperfect inscription, in which although the actual extremity of the planetary sphere is not touched by the centers of the faces of the figure, yet at least something inside it is, namely the mean between the spheres of aphelion and perihelion, if that is possible by other arguments. On the other hand, the dodecahedron and its ally have inscribed spheres which are definitely inexpressible both in the length of the semidiameter and in the square. Therefore, they ought to represent inscription which is definitely imperfect, and touches nothing whatever of the planetary sphere, that is, falling short, and definitely not making contact up to the sphere of aphelion of the planet with the centers of its faces. Although the hedgehog is akin to the dodecahedron and its ally, yet it has some similarity to the tetrahedron. For the semidiameter of the sphere inscribed in its inverted edges[159] *is indeed incommensurable with the semidiameter of the circumscribed sphere, but instead it is commensurable in length with the distance between neighboring pairs of vertices.*[160] *Thus the perfection of commensurability of the radii is almost as great as in the tetrahedron, but in the other respect its imperfection is as great as in the dodecahedron and its ally. It is fitting, therefore, that the inscription agrees with that physically, and is neither definitely tetrahedric, nor definitely dodecahedric, but of an intermediate kind. Hence because the tetrahedron ought to have extended with its faces to the outer surface of the sphere,*[161] *but the dodecahedron should have failed to reach it by a certain distance, as it is, this spiked figure stands with its inverted edges between the icosahedron's space and the outer surface of the inscribed sphere, very nearly reaching the latter's outer surface—if nevertheless even that figure should have been received into the fellowship of the other five, and its laws could have been tolerated by their existing laws. Yet what am I saying, "could have been tolerated"?—which they could not do without. For if inscription which was lax and not touching agreed with the dodecahedron, what else could restrain that unlimited laxity within the bounds of a definite amount but this subsidiary figure, akin to the dodecahedron and icosahedron, with its inscription very nearly touching, and falling short (if however it does fall short) no more than the tetrahedron is in excess and penetrates? We shall now discuss that amount in what follows.*

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[159] By this description Kepler means the sphere which passes through the midpoints of the edges of the dodecahedron forming the nucleus of the hedgehog.

[160] See note 22.

[161] That is, the sphere should have touched the tetrahedron at the centers of its faces.

This reason drawn from the association of the hedgehog with the two figures which are akin to it (that is to say, for the determination of the proportion of the spheres of Mars and Venus, which they had left indefinite) is rendered very probable by the fact that the semi-diameter of the sphere of the Earth, 1000, is found to be very close to the mean position proportionally between the sphere of the perihelion of Mars and that of the aphelion of Venus, as if the space which the hedgehog claims for the figures which are akin to it were divided proportionally between them, in virtue of their similarity.[162]

See an illustration of the hedgehog in Bk. V, p. 398 where *PP* are the solid angles, *OO* the intermediate inverted edges.

XLVIII. Proposition

There was not pure liberty for the inscription of the regular solid figures between the planetary spheres; for it was impeded over small details by the harmonies set up between the extreme motions.[163]

For by Axioms I and II the proportion of the spheres of each figure ought not to have been expressed immediately by itself, but through it there had first to be sought, and fitted to the extreme motions, harmonies which were very closely related to the actual proportions of the spheres.

Next, so that by Axioms XVIII and XX there could be universal harmonies of the two kinds, it was necessary to add some leaven to the greater harmonies of individual pairs, by the a posteriori *arguments. Therefore, to make it possible for these to stand and to depend on their own arguments, there were required intervals a little at variance with those drawn from perfect inscription of the figures between the spheres, by the laws of the motions expounded in Chapter III. To prove that, and to show how much departure there is in individual figures through the harmonies confirmed by appropriate arguments, come, let us derive from them the distances of the planets from the Sun, by a new form of calculation not before attempted by anyone.*[164]

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[162] With this proposition Kepler justified the discrepancy between the actual planetary distances and the interpolation of the dodecahedron and icosahedron, without appealing to the requirements of the harmonies. The introduction of the hedgehog, providing an *a priori* explanation of the differences, thus constitutes a modification of the original polyhedral hypothesis published in the *Mysterium cosmographicum*. Cf. note 37.

[163] Although Kepler had been able to explain the gap between Mars and Venus by means of his modified polyhedral hypothesis, other discrepancies could only be explained in terms of the requirements of the planetary harmonies, which in his view had to take precedence. Cf. note 23.

[164] Although Kepler's purpose, as stated here, is to compare the distances derived from the harmonic theory with those derived from the interpolation of the regular polyhedra between the planetary spheres, he is also preparing an empirical confirmation of the harmonic theory in accordance with his methodological principle that "hypotheses must be based upon and confirmed by observations." For the planetary distances from the sun, which he proposes to derive from the harmonic theory, may be compared directly with those derived from the observations of Tycho Brahe.

Calculation of the planetary distances as they emerge from the harmonic constitution.

Now there will be three heads of this enquiry: First, from the two extreme motions of each planet will be sought the distances, similarly extreme, of it and the Sun, and from them the semidiameter of the sphere measured by the extreme distances as appropriate to each planet. Second, from the same extreme motions, measured in the same units in all cases, will be sought the mean motions, and their proportion. Third, from the proportion of the mean motions now revealed the proportion of the spheres or of the mean distances will be investigated, and along with it that of the extremes also; and that will be compared with the proportions taken from the figures.

As far as the first is concerned: it must be recalled from Chapter III, number VI, that the proportion of the extreme motions is the square of the inverse proportion of the corresponding distances from the Sun. Therefore, since the proportion of squares is the square of the proportion of their sides, then the numbers by which the extreme motions are expressed will be considered as square, and finding their roots will give the extreme distances. It is easy to take the arithmetic mean of those for finding the semidiameter of the sphere and the eccentricity. Therefore, the harmonies so far confirmed have prescribed as follows:[165]

For Planets.	*Proportions of motions.*	*Roots of proportions either augmented or as roots of multiples.*	*Hence semidiameter of sphere.*	*Eccentricity.*	*Measured with semidiameter of sphere as 100000.*
For Saturn by XXXVIII.	64 81	80 90	85	5	5882
For Jupiter by XXXVIII.	6561 8000	8100 89444	85222	4222	4954
For Mars by XLI.	25 36	50 60	55	5	9091
For the Earth by XXVIII.	2916 3125	93531 96825	95178	1647	1730
For Venus by XXVIII.	243 250	9859 10000	99295	705	710
For Mercury by XLV.	5 12	63250 98000	80625	17375	21551

For the second of the proposed heads we again have need of Number XII of Chapter III, where it was shown that the number which expresses the mean motion in the proportion of the extremes is less than their arithmetic mean, and less also than their geometric mean, by half the difference between the two means.[166] *And because we are seeking for all the mean motions measured in*

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[165] The values for the extreme distances of the individual planets given in column 2 of the table are calculated using the formula $M_a/M_p = R_p^2/R_a^2$. See note 41.

[166] From the proportions of the motions of the individual planets given in column 3, the mean motions (column 6) are calculated using the formula $m = G - \frac{1}{2}(A - G)$. See note 48. From the harmonies established between pairs of planets (col-

the same units, as all the proportions which have so far been confirmed between pairs of motions, and also all the planets' own motions, are set out in the common measure of their lowest common factor, then we should seek for the arithmetic mean by halving the difference between the extreme motions in each case, and the geometric by multiplying one extreme by the other and extracting the root of the product. Then by subtraction of half the difference of the means from the geometrical mean the numerical value of the mean motion will be established, in each of the extreme motions' own measure, which is easily transformed into the common measure by the rule of proportions.

Harmonic Proportions of Pairs	*Values of Extreme Motions*		*Individual planets' own props.*	*Means of individual motions continued* *Arithmetic*	*Geometric*	*Halves of difference*	*Value of mean motion in own measure*	*common measure*
1	Saturn	139968.	64	72.50.	72.00.	25.	71.75.	156917.
1	Saturn	177147.	81					
30 2	Jupiter	354294.	6561	7280.5.	7244.9.	178.	7227.1.	390263.
5	Jupiter	432000.	8000					
24	Mars	2073600.	25	30.50.	30.00.	25.	29.75.	2467584.
2	Mars	2985984.	36					
32 3	Earth	4478976.	2916	3020.500.	3018.692.	904.	3017.788.	4635322.
5	Earth	4800000.	3125					
5	Venus	7464960.	243	246.500.	246.475.	125.	246.4625.	7571328.
1 3 8	Venus	7680000.	250					
5	Mercury	12800000.	5	8.500.	7.746.	377.	7.369.	18864680.
4	Mercury	30720000.	12					

The figures after the full stop relate to the accuracy of the number in decimal parts

Then from the prescribed harmonies has been found the proportion of the mean daily motions, that is to say the proportion between the values of each, in degrees and minutes, and so on, compared with each other; and it is easy to examine how closely it comes to astronomy.[167]

The third head of what has been proposed needs Number VIII of Chapter III. For having found the proportion of the mean daily motions in individual planets, we can find the proportion of their spheres as well. For the proportion of the mean motions is the sesquialterate of the inverse proportion of the spheres. But the proportion of cubic numbers is also the sesquialterate of the proportion of the squares belonging to the same roots in the table of Clavius, which he appended to his Practical Geometry. Hence, if our values for the mean motions

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umn 1) and the proportions of the motions of the individual planets (column 3), values are now calculated in the same scale for the extreme motions of all the planets (column 2). By increasing the values for the mean motions already calculated (column 6) in the proportion of the corresponding numbers in columns 2 and 3, the mean motions are finally given in the same scale. The slowest motion (that of Saturn at aphelion) is taken as $139968 = 3^7.2^6$ in order to avoid fractions. It may be noted, however, that decimal fractions occur in columns 4, 5, and 6.

[167] Kepler has not actually given the theoretical mean daily motions in degrees and minutes (surely he intended to say minutes and seconds), but only in an arbitrary common scale.

are sought (curtailed, where necessary, to an equal number of figures) among the cubes in that table, they will show to their left under the heading of the squares the values for the proportion of the spheres. Then the eccentricities ascribed above to the individual planets, in the appropriate measure of the semidiameters of each, will easily be transposed by the rule of proportions into the measure which is common to them all; and hence those values added to the semidiameters of the spheres, and subtracted from them, will establish the extreme distances of the individual planets from the Sun. However, we shall give for the semidiameter of the sphere of the Earth the round measure 100,000, as is customary in astronomy, with the intention that this number, either squared or cubed, always consists of pure zeros. Thus we shall also bring the mean motion of the Earth out to the number 1,000,000,000, working by the rule of proportions to make the value for the mean motion of each planet to the value for the mean motion of the Earth as 1,000,000,000 is to the new measure. So the matter can be carried through with only five cube roots, by comparing them individually with the single value for the Earth.[168]

	Values from the mean motions in		*Values of proportion of spheres found among squares.*	*Semi-diameters as above.*	*Eccentricity in*		*Resulting extreme distances*	
	original measure	*new inverse measure to be sought among cubes*			*own measure as above.*	*common measure.*	*at aphelion*	*at perihelion.*
Saturn.	156917.	29539960.	9556.	85.	5.	562	10118.	8994
Jupiter.	390263.	11877400.	5206.	85222.	4222.	258	5464.	4948
Mars.	2467584.	1878483.	1523.	55.	5.	138	1661.	1384
Earth.	4635322.	1000000.	1000.	95178.	1647.	17	1017.	983
Venus.	7571328.	612220.	721.	99295.	705.	5	726.	716
Mercury.	18864680.	245714.	392.	80625.	17375.	85	476.	308

In the last column, therefore, may be seen what numbers emerge to express the converging distances of the pairs of planets; and all of them approach very closely the distances which I have found from the observations of Brahe.[169] *In the case of Mercury alone there is a tiny difference. For astronomy seems to give it the distances 470, 388, and 306, which are all shorter. It seems reasonable to guess that the reason for the discrepancy is either in the paucity of observations or in the size of the eccentricity. See Chapter III. But I am hurrying on to the end of the calculation.*

For it is now easy to compare the proportion of the spheres of the figures with the proportion of the converging intervals.

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[168] The numbers in column 2 are obtained by dividing those of the first column into 4635522 × 1000000. Then taking the square of the cube root and dividing by 10 gives the numbers in column 3, which are taken to represent the mean distances. In the case of Mars, the mean distance should be 1522. The eccentricity of Mercury should be 84 and this is in fact the value used to calculate the aphelion and perihelion distances. The aphelion distance of Mars should be 1660.

[169] The distances derived from Tycho Brahe's observations are given, in the same scale, in the table of comparison of the distances with the harmonies in Chapter IV.

For if the semidiameter of the sphere circumscribed round a figure, which in the common measure is 100000, were to be		Then the semidiameter of the inscribed sphere for	instead of	will become	Whereas the distance from the harmonies is	
In the case of the cube	8994	Saturn	57735	5194	mean of Jupiter	5206
In the case of the tetrahedron	4948	Jupiter	33333	1649	aphelion of Mars	1661
In the case of the dodecahedron	1384	Mars	79465	1100	aphelion of Earth	1018
In the case of the icosahedron	983	Earth	79465	781	aphelion of Venus	726
In the case of the hedgehog	1384	Mars	52573	728	aphelion of Venus	726
In the case of the octahedron	716	Venus	57735	413	mean of Mercury	392
In the case of the square of the octahedron	716	Venus	70711	506	aphelion of Mercury	476
or	476	Mercury	70711	336	perihelion of Mercury	308

That is, the cubic faces descend a little below the mean distance of Jupiter; the octahedric faces do not descend absolutely to the actual mean distance of Mercury; the tetrahedric faces descend a little below the greatest distance of Mars; the edges of the hedgehog do not descend absolutely to the actual greatest distance of Venus; but the faces of the dodecahedron fall far short of the distance at aphelion of the Earth; the faces of the icosahedron also fall far, and almost proportionately short of the distance at aphelion of Venus; and last, the square of the octahedron definitely does not fit at all.[170] *But there is no harm in that; for what has a plane figure to do with solids? You see therefore that if the distances of the planets are deduced from the harmonic proportions of the motions, which have been demonstrated so far, the former must necessarily turn out to be of the size which the latter permit, but not of the size which the laws of free inscribing would require, as prescribed in Proposition XLV. For this "geometrical cosmos" of perfect inscription was no longer close to that other "possible harmonic cosmos," to use Galen's words taken from the frontispiece of this Book V. That much had to be demonstrated from actual numerical calculation for the elucidation of the proposition in question.*

I do not disguise the fact that if I were to increase the harmony of the converging motions of Venus and Mercury by Venus' motions' own proportion, and in consequence to diminish Mercury's own harmony by the same amount, then I should obtain by that procedure as the distances of Mercury and the Sun 469, 388, and 307, which are very precisely those indicated by astronomy. But, first, I cannot defend that diminution by harmonic arguments, for the motion of Mercury at aphelion will not agree with any musical scale; and in planets which are opposed in the world a complete pattern of opposition is not maintained in all properties. Second, the mean daily motion of Mercury is made too great, and so the periodic time of Mercury, which is the most certain in the whole of astronomy, is shortened too much. Thus

[170] In the *Mysterium cosmographicum*, Chapter 13, Kepler found a better fit by taking the circle in the octahedron-square instead of the inscribed sphere as the outer boundary of the orbit of Mercury, and he gave *a priori* reasons for this choice. See Duncan (1981), pp. 153 and 173.

I stand by the harmonic commonwealth of the motions which has been assumed here and confirmed by the whole of Chapter IX. Nevertheless, by this example I challenge as many of you as will chance to read this book and are imbued with the disciplines of mathematics and knowledge of the highest philosophy: come, be vigorous and either tear up one of the harmonies which have been everywhere related to each other, change it for another one, and test whether you will come as close to the astronomy laid down in Chapter IV; or else argue rationally whether you can build something better and more appropriate on to the heavenly motions, and overthrow either partly or wholly the arrangement which I have applied. Whatever contributes to the glory of Our Founder and Lord is equally to be permitted to you throughout this my book; and I have assumed that I myself am permitted up to this hour freely to change anything which I could discover which was incorrectly conceived in the preceding days if my attention nodded or my enthusiasm was hasty.

Challenge.

IL. Envoi

It was right that in the genesis of the distances the solid figures should give way to the harmonic arguments, and the greater harmonies of the pairs to the universal harmonies of all, to the extent to which the latter was necessary.

By a splendid coincidence we have come to the square of the sevenfold, 49, so that like a kind of Sabbath it may succeed the foregoing six solid octaves of utterances on the structure of the heavens. Also I justly made into an envoi what could have been put earlier among the Axioms; for God also when He had now completed the task of Creation "saw all that He had done, and behold! it was very good."[171]

There are two limbs of the envoi. The first, on the harmonies in general, is demonstrated as follows. For where there is a choice between different things which do not allow each other to have sole possession, in that case the higher are to be preferred, and the lower must give way, as far as is necessary, as the actual word "cosmos," which means "decoration," seems to bear witness. But harmonic decoration is as far above the simple geometrical as life is above the body, or form above matter.

See Book IV, page 367.

For just as life completes the bodies of animate beings, because they were born to lead it, which follows from the archetype of the world, which is the actual divine essence, so motion measures out the regions allotted to the planets, to each its own, as a region has been assigned to a star so that it could move. But the five solid figures, in virtue of the word itself, relate to the spaces of the regions, and to the number of them and of the bodies; but the harmonies to the motions. Again, as matter is diffuse and unlimited in itself, but form is limited, unified, and itself the boundary of matter; so also the number of the

See Book IV.

[171] Genesis I, v. 31.

geometrical proportions is infinite, the harmonies are few. For although even among the geometrical proportions there are definite degrees of limitation and shape and restriction, and not more than three can be formed by the ascription of spheres to regular figures, yet even to these is attached an accidental property in common with all the rest, that is the presupposition of an infinite possible division of quantities. Those of which the terms are incommensurable with each other also involve that in practice in a way. But the harmonic proportions are all expressible, and the terms of all of them are commensurable. Also, they have been taken from a definite and limited class of plane figures. Now infinite divisibility signifies matter, but commensurability or expressibility of term signifies form. Therefore, as matter strives for form,[172] *as a rough stone, of the correct size indeed, strives for the Idea of the human form, so the geometrical proportions in the figures strive for harmonies; not so as to build and shape them, but because this matter fits more neatly to this form, this size of rock to this effigy, and also this proportion in a figure to this harmony, and therefore so that they may be built and shaped further, the matter in fact by its own form, the rock by the chisel into the appearance of an animate being, but the proportion of the spheres of the figures by its own, that is, by close and fitting harmony.*

What has been said up to this point will be made clearer by the story of my discoveries. When, twenty four years ago, I had engaged in this study, I first enquired whether the individual circles of the planets were separated by equal distances from each other (for in Copernicus the spheres are separated, and do not mutually touch each other). Of course I acknowledged nothing as more splendid than the relationship of equality. However, it lacks a head and a tail, for this material equality provided no definite number for the moving bodies, no definite size for the distances. Therefore, I thought about the similarity of the distances to the spheres, that is about their proportion. But the same complaint followed, for although in fact distances between the spheres emerged which were certainly unequal, yet they were not unequally unequal, as Copernicus would have it, nor was the size of the proportion nor the number of the spheres obtained. I moved on to the regular plane figures: they produced the distances in accordance with the ascription of their circles, but still in no definite number. I came to the five solids:[173] *in this case they revealed both the number of the bodies and nearly the right size for the intervals—so much so that I appealed over the remaining discrepancy to the state of accuracy of astronomy. The accuracy of astronomy has been perfected in the course of twenty years; and see! there was still a discrepancy between the distances and the solid figures, and the reasons for the very unequal distribution of the eccentricities among the*

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[172] As is clear from the explanation that follows, Kepler does not ascribe to matter an active force that enables it to achieve form. On the contrary, it is the form that shapes matter, but the matter has a certain adaptability or aptness to receive the form.

[173] Kepler has here indicated very briefly that the polyhedral hypothesis was not the result of his first attempt to relate the planetary distances. A fuller account of his earlier trials is given in the preface of the first edition of the *Mysterium cosmographicum*. See Duncan (1981), pp. 62–69.

planets were not yet apparent. Of course in this house of the cosmos I was looking for nothing but the stones—of more elegant form, but of a form appropriate to stones—not knowing that the Architect had shaped them into a fully detailed effigy of a living body. So little by little, especially in these last three years, I came to the harmonies, deserting the solid figures over fine details, both because the former were based on the parts of the form which the ultimate hand had impressed, but the figures from matter, which in the cosmos is the number of the bodies and the bare breadth of their spaces, and also because the former yielded the eccentricities, which the latter did not even promise. That is to say, the former provided the nose and eyes and other limbs of the statue, for which these latter had only prescribed the external quantity of bare mass.

Hence just as the bodies of animate beings have not been made, and a mass of stone is not usually made, according to the pure norm of some geometrical figure, but something is removed from the external round shape, however elegant (though the correct amount of bulk remains) so that the body can take on the organs necessary to life, and the stone the likeness of an animate being, similarly also the proportion which the solid figures were to prescribe for the planetary spheres, as lower, and having regard only to a body of a particular size and to matter, must have given way to the harmonies, as much as was necessary for the former to be able to stand close and to adorn the motions of the globes.

Partly assuming No. XVIII of the Axioms above.

The other limb of the envoi, on universal harmonies, has a proof which is akin to the previous one. For that which chiefly makes the world perfect ought preferably to have the supreme hand in perfection; but in return it is the one from which something must be removed (if something must be removed from one or the other), because in this case it is in the secondary position. But it is the universal harmony of all which chiefly makes the world perfect, rather than the individual twinnings of neighboring pairs. For harmony is a certain relationship of unity: therefore they are united if they are all at one at the same time rather than if each pair separately agree in pairs of harmonies. So that in a conflict between the two, one or other of pairs of harmonies of the pairs of planets must have yielded so that the universal harmonies of all could stand. However, the greater harmonies of the diverging motions must have yielded, rather than the lesser harmonies of the converging motions. For if the former are diverging, therefore they are having regard not to the planets of the pair specified, but to other neighboring ones; and if the latter are converging, therefore the motion of one planet is tending towards the motion of the other, in exchange. So in the pair of Jupiter and Mars, the motion of the former at aphelion tends towards Saturn, and of the latter at perihelion towards the Earth; but the motion of the former at perihelion tends towards Mars, and that of the latter at aphelion towards Jupiter. Therefore, the harmony of the latter motions belongs more particularly to Jupiter and Mars; and the harmony of the former, diverging, motions is in a way more alien from Jupiter and Mars. Now the relationship of union which binds together pairs and neighboring pairs was less damaged if the harmony which is more alien and more remote from them were leavened, than if the harmony which belongs particularly to them were, that is the harmony which exists between the more closely neighboring motions of neighboring planets. However, this leavening was not very great. For the relationship was *found, by which both the universal harmonies of all the planets would stand, and those of two distinct kinds and with a certain latitude in tuning, which would equal at least a comma, and also the two individual harmonies of neighboring pairs of planets would be protected: in fact perfect harmonies of the converging motions in four pairs; similarly perfect harmonies of the motions at aphelion in one, in perihelion in two; but of the diverging intervals in four pairs, within a discrepancy of one diesis, the smallest interval, by which the human voice in figured melody is almost perpetually out of tune. However, in the single case of Jupiter and Mars the discrepancy is between a diesis and a semitone. It is therefore evident that this mutual concession on all sides holds exceedingly good.*

So far, therefore, we have "delivered our *envoi*" on the work of God the Creator. It now remains for me, at the very last, to take my eyes and hands away from the table of proofs, lift them up to the heaven, and pray devoutly and humbly to the Father of light:

O Thou who by the light of Nature movest in us the desire for the light of grace, so that by it thou mayest bring us over into the light of glory; I thank Thee, Creator Lord, because Thou hast made me delight in Thy handiwork, and I have exulted in the works of Thy hands. Lo, I have now brought to completion the work of my covenant, using all the power of the talents which Thou hast given me. I have made manifest the glory of Thy works to men who will read these demonstrations, as much as the deficiency of my mind has been able to grasp of its infinity. My intellect has been ready for the most accurate details of philosophy. If anything unworthy of Thy intentions has been put forward by me, miserable worm that I am, born and nourished in a slough of sins, which thou wouldst wish men to know, inspire me also to set it right; if I have been enticed into temerity by the wonderful splendor of Thy works, or if I have loved my own glory among men, while advancing in work destined for Thy glory, mildly and mercifully pardon it; and last, be gracious and deign to bring about that these my demonstrations may be conducive to Thy glory and to the salvation of souls, and may in no way obstruct it.

CHAPTER X.

Conjectural Epilogue on the Sun

From the heavenly music to the hearer; from the Muses to Apollo the choirmaster; from the six planets which go around and make the harmonies, to the Sun at the center of all the orbits, motionless in his place, but revolving on his own axis. For whereas there is the most complete harmony between the extreme motions of the planets, not in the sense of the true speeds through the ethereal air, but in the sense of the angles which are formed by the ends of the daily arcs of the planets' orbits, joined to the center of the Sun, yet the harmony does not ornament the ends, that is, the individual motions, in themselves, but insofar as they are linked and compared with each other and are made the object of some mind; and since no object is arranged vainly, and without something which is moved by it, those angles seem in fact to presuppose some agency, like our sight, or certainly the sensation of it, on which see Book IV. Sublunary nature perceived the angles formed at the Earth by the radii from the planets. Indeed for dwellers on the Earth it is not easy to conjecture what sort of vision, what eyes, there may be on the Sun, or what other instinct for perceiving these angles, even without eyes, and of estimating the harmonies of the motions which enter the forecourt of the mind by whatever door, indeed what Mind there is on the Sun. Yet however that may be, certainly this positioning of the six primary spheres round the Sun, honoring him with their perpetual revolution, and, so to speak, adoring him (just as Jupiter's globe separately by four moons, Saturn by two moons, and the Earth and we its inhabitants by a single moon are girded by their course, honored, cherished, and served) and this special matter of the harmonies which is now added to that consideration, a very clear trace of the highest providence in the affairs of the Sun, wrings from me the following confession. Not only does light go out to the whole world from the Sun, as from the focus or eye of the world, as all life and heat does from the heart, all motion from the ruler and mover; but in return there are collected at the Sun from the whole cosmic province, by royal right, these, so to speak, repayments of the most desirable harmony, or rather images of the pairs of motions flowing to it are linked together into a single harmony by the working of some mind, and so to speak stamped into coin out of rough silver and gold; and lastly in the Sun is the Senate, Palace, and Government House or Court of the whole kingdom of nature, whatever Chancellors, Princes, or Prefects the Creator has given to it, and everything with which He has provided those seats, whether it was created by Him straight away at the beginning or whether they had at some time to be transported thither. For that terrestrial embellishment also for the chief part of itself lacked the investigators and appropriators, for which it was nevertheless destined, and the seats were vacant. Therefore, the enquiry steals into the mind, what did the ancient Pythagoreans mean in Aristotle[174] when they used to call the center of the world (to which they referred the Fire, though their underlying meaning was the Sun) the sentry of Jupiter ([in Greek] "the Guardian of Zeus"); and what was the ancient translator[175] turning over in his mind, when he rendered the verse in the Psalm as "In the Sun He has placed His tabernacle." But I have also just chanced on the hymn of Proclus the Platonic[176] philosopher, written to the Sun, and packed with venerable secrets, if you remove from it the one phrase "Hear me"; though the ancient translator, already quoted, excused that very phrase in it to a certain extent, as when of course he is invoking the Sun he takes as the underlying meaning, "He Who placed His tabernacle in the Sun." For Proclus lived in the time when it was a crime to proclaim Jesus of Nazareth our Savior as God and despise the gods of the gentile poets, which was punished with every torment by the monarchs of this world and indeed the people themselves.[177] Therefore, Proclus, who even by his own Platonic philosophy had perceived the Son of God from far off by the natural light of his own mind as the true brightness which comes into this world and illuminates every man, who already knew that it was in vain to seek divinity along with the superstitious populace, yet preferred to seem to seek God in the Sun than in Christ the living man. Thereby he at the same time both deceived the gentiles by honoring the Titan of the poets in words alone; and served his own philosophy, with the intention of drawing both the gentiles and the Christians away from sensible things, the former from the visible Sun, the latter from the Son of Mary, as he rejected the mystery of the incarnation, trusting too much in the natural light of his mind; and lastly, what the Christians held as most divine, and most in agreement with the Platonic philosophy, he took over from them and adopted into his own philosophy. Thus to the doctrine of the Gospel of Christ,

Which is often mentioned in the earlier books.

Under Constantine, Maxentius, and Julian the Apostate.

The judgement of the ancients on his book "*The Temple of the Mother of God*" was that in it the universal doctrine about God was set out with a certain divine rapture, and the author's many tears which were evident in it took away all suspicion from his audience. Yet the same man wrote 18 *Theses* against the Christians which John Philoponus opposed, criticizing his ignorance of Greek matters which he had nevertheless undertaken to defend.[178]

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174 Aristotle, *De caelo*, 293 b 1–6.

175 The old Bible-translator is the editor of the Vulgate text. See Psalm 19.4, in the King James version (= 18.6 in the Vulgate). Kepler's rendering is not an accurate translation of the original Hebrew.

176 This hymn of Proclus was printed in Kepler's time as an addition to those of the Orphic tradition. See *Musaei opusculum de Herone et Leandro, Orphei Argonautica, eiusdem Hymni, Orpheus de lapidibus* (Venice, 1517), 63–64. For an English translation, see T. Taylor, ed., *The Hymns of Orpheus* (London, 1793), 122–123.

177 Kepler is mistaken, for the emperors he cites reigned in the fourth century whereas Proclus lived in the fifth century. In the time of Proclus, Christianity was the prevailing religion in the Roman Empire.

178 The book is lost. However, the 18 theses against the Christians are preserved in J. Philoponus, *De aeternitate mundi contra Proclum*, 1535, Ed. H. Tabe (Leipzig, 1899).

contrary to this hymn of Proclus, is granted activity for its own purpose. Let this Titan have what belongs to him, the "golden reins" and "the storehouse of light, the midmost seat in the ether, the brilliant circle at the heart of the cosmos," an appearance which Copernicus also bestows on him. Let him also have his "returning chariot drivers," though he does not have them among the ancient Pythagoreans, but instead of them he has "the center," "the guardian of Zeus" (a dogma of theirs which, distorted by the oblivion of the centuries, as if by a flood, was not recognized by their successor Proclus) and "offspring sprouting" from himself, and everything natural. In return let the philosophy of Proclus give up to Christian doctrines, let the sensible Sun give up to the Son of Mary, that Son of God whom Proclus addresses under the name of Titan, of "sustainer of life, king, holder of the key of the fountain," and in the words "thou who hast filled all things with thy providence that stirs the soul"; and that immense power of destiny, and what was read in no* philosophy before the promulgation of the Gospel, the demons that dread his threatening scourge, the demons that lie in wait for souls, "so that they may forget the brilliant court of the father on high"; and who but the word of the Father is the "image of the god who is father of all, on whose appearance from the begetter who must not be named the din of the clashing elements ceased" according to the saying "The Earth was a rough and formless mass, and there was darkness over the face of the abyss," and "God divided the light from the darkness, the waters from the waters, and the sea from the dry land," and "all things were brought about by the word itself"? Who but Jesus the Nazarene, the Son of God, "the bringer up of spirits," the shepherd of souls, to Whom must be offered "the prayer of many tears," that He may purge us of our sins and snatch us out of the filth of "the offspring" (as if He admitted the kindling of original sin), and guard us from punishments and evils, "making mild the swift eye of justice," that is to say the anger of the Father? And what else do we read, as if taken from the hymn of Zacharias,[180] scattering the man-destroying, venom-born mist," that is to say that when souls are in the midst of darkness and the shadow of death He gives us the "holy light" and "unshakable bliss from beautiful reverence": that is, to serve God in holiness and justice all our days.

Of course Proclus had disguised what did not support his philosophy. Was that hymn, then, part of The Temple of the Mother Goddess?

* However, several similar things are attributed in Suidas[179] to Orpheus who lived in very ancient times and was almost contemporary with Moses, as if he was the latter's disciple. See also the *Hymns* of Orpheus, on which Proclus wrote.

Therefore, let us now set aside this and similar matters, and relinquish them to the doctrine of the Catholic Church, to which they properly belong; but let us now look at the chief reason why the Hymn has been mentioned. For the same Sun which "draws off the rich flow

[179] The lexicon of Suidas, written between 1000 and 1150, is a principal source for our knowledge of the old philosophers. The hymns known under the name of Orpheus belong to a much later time than the Orpheus who lived in the early period of Greek history. Moses, who lived about 1500 BC, is repeatedly confused with Musaios, a member of the mythical circle around Orpheus.

[180] Luke 1. 68–80.

of harmony from on high," the same from whose stock Phoebus sprang, and "playing wondrously with his lute lays to rest the great surge of his loud-roaring offspring," and whose partner in the chorus is Apollo Paean, "who filled the broad cosmos with harmony that takes away pain," the same Sun, I say, is saluted in the very first line of the Hymn as "king of the intellectual fire." By this beginning he indicates what the Pythagoreans understood by the word "fire" (so that it is remarkable that the disciple disagreed with the masters on the position of the center, which they gave to the Sun itself), and at the same time transfers his whole hymn from the body of the Sun, and from its nature by daylight, which are sensible, to intellectual things; and he has assigned the royal seat on the body of the Sun to that "intellectual fire" of his (perhaps the creative fire of the Stoics), that created God of his own Plato, his chief Mind or "pure intellect," confusing together the creation and Him through Whom all things were created. But we Christians, taught to distinguish better the eternal and uncreated Word, which was "with God,"[182] and which is not kept in any abode, though it is within all things, excluded from none, and though it is itself outside all things, know that the flesh was taken up from the womb of the most glorious Virgin Mary into the unity of the Person, and that when the ministry of the flesh was completed He occupied as His royal seat the heavens, in which the heavenly Father also is acknowledged to dwell, as a part of the world which in some way excels all others, that is in glory and majesty, and that He also promised to His faithful dwelling places in that house of His Father. For the rest we consider it to be quite useless to enquire after any further detail about that seat, and to summon the natural senses or reason to hunt out what the eye has not seen, the ear has not heard, and what has not ascended into the heart of man. Indeed we deservedly subject the created mind, however great its pre-eminence may be, to its Creator; and we do not introduce Intelligences as gods with Aristotle and the gentile philosophers, nor innumerable armies of planetary spirits with the Magi, and we do not put them forward to be adored or to be stirred up by magic superstitions to carry on mutual intercourse. Taking careful precautions against that, we freely enquire what the nature of each mind may be, particularly if in the heart of the world it plays the part of the soul of the world, and is more tightly tied to the nature of things (or even if some intelligent creatures, of different nature from the human, happen to inhabit a globe which is in that way animated, or will inhabit it). But if we may follow the thread of analogy and pass through the labyrinths of the mysteries of nature, it would not, I think, be absurd for someone to argue that the disposition of the six spheres towards their common center, and therefore the center of the whole world,

So also Orpheus[181] "drawing out the enharmonic course of the cosmos."

See Chapter XXIV of my book *On the New Star*, on the soul of the world and some of its functions.

[181] See *Musaei opusculum* (note 61), 43.
[182] John 1.1.

is the same as that of "thought" to "mind," as these faculties are distinguished by Aristotle, Plato, Proclus, and others; and again, that the disposition of the local revolutions of the individual planets round the Sun towards the Sun's "immutable" gyration in the central space of the whole system is the same as the disposition of the "thinkable" to the "understandable," of the manifold processes of reasoning towards the completely simple understanding of the mind. For as the Sun in its revolution about its own axis moves all the planets by the emanation which it sends out from itself, so also the mind, as the philosophers tell us, understanding itself and all that is in itself, stimulates the use of reason, and by spreading and unfolding its simplicity causes all things to be understood. And so closely are the motions of the planets round the Sun and the processes of reasoning linked and tied to each other that if the Earth, our home, did not measure out its annual circuit in the midst of the other spheres, changing place for place, position for position, human reasoning would never struggle to the absolutely true distances of the planets, and to the other things which depend on them, and would never establish astronomy. On the other hand, the counterpart of the immobility of the Sun at the center of the world, by a beautiful symmetry, is the simplicity of the understanding, because hitherto we have always taken for granted that those solar harmonies of the motions are not delimited either by the difference of directions or by the breadth of the spaces of the world. Of course if any mind is viewing those harmonies from the Sun, it lacks the props of motion and different positions for his seat, to which to bind the reasoning and the reflection necessary for measuring out the distances of the planets. Therefore, it compares the daily motions of each planet, not as they are in their orbits, but as they glide through their angles at the center of the Sun. Thus if it has knowledge of the size of the spheres, that must necessarily belong to it *a priori*, without the labor of reasoning; and it has been made clear above from Plato and Proclus that that is true to a certain extent both of human minds and of sublunary nature.

To which the spots on the Sun bear witness. This is demonstrated in the *Commentaries on the Motion of Mars*.[183]

See Book IV, page 301.

See *The Optical Part of Astronomy*, Chapter IX.

That being the case, it would not in fact be surprising if someone who was heated by taking too liberal a draught from this cup of Pythagoras, which Proclus pledges straight away in the very first verse of his Hymn, if he was put to sleep by the delightful harmony of the chorus of planets, were to begin to dream that among the other globes passing from place to place round the Sun were disseminated reflective or reasoning faculties, of which that on the middle one of those globes, that is on the human Earth, must be reckoned certainly the most outstanding and most absolute, whereas on the Sun dwells simple Understanding, the "intellectual fire" or "mind," the fount of all harmonies, whatever it may be.

Grant me license to imitate both Plato's Atlantis in story-telling and Cicero's Scipio in dreaming.

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[183] *Astronomia nova*, Chapter 34.

For if Tycho Brahe, considering the bare immensity of those globes, believed that it did not exist pointlessly in the world, but was packed with inhabitants, how much more convincing will it be for us, perceiving the variety of the works and intentions of God on this globe of Earth, to adopt a conjecture about the others as well? For He has created species to inhabit the waters, though there is no place under them for air, which living things draw in; He has sent into the immensity of the air birds propped up by feathers; He has given to the snowy tracts of the north white bears and white foxes, and as food the monsters of the sea to the one, the eggs of the birds to the other; He has given lions to the deserts of burning Libya, camels to the far spread plains of Syria, and endurance of hunger to the one, of thirst to the other. Has He then used up all His skill on the globe of the Earth so that He could not, or all His goodness, so that He would not wish to adorn with suitable creatures other globes also, which were commended either by the amplitude or brevity of their revolutions, or the nearness and remoteness of the Sun, or the difference of their eccentricities, or the brightness or dimness of their bodies, or the properties of the figures on which every region is supported? For see! as the generations of living creatures on this globe of the Earth have an image of the masculine in the dodecahedron, of the feminine in the icosahedron (the former of which supports the sphere of the Earth on the outside, the latter on the inside), and lastly an image of procreation in the divine proportion of that marriage and in its inexpressibility, what shall we suppose the other globes have from the other figures? For whose benefit do four moons gird Jupiter, and two Saturn, in their courses, as this single Moon of ours does our home? In fact we shall also reason in the same way about the globe of the Sun, and we shall, so to speak, incorporate conjectures drawn from the harmonies, and the rest, very weighty by themselves indeed, with other conjectures, tending rather towards the bodies, and fitted rather for catching the crowd. Is that globe empty, but the others full, if everything else corresponds more closely? if just as the Earth breathes out clouds, the Sun breathes out black soot? if just as the Earth is moistened and grows green with rains, the Sun grows bright with those burning spots springing out from its body, which is wholly on fire, with brighter flamelets? What use is this furnishing, if the globe is empty? Do not the very senses themselves cry out that fiery bodies inhabit it, which have the capacity for simple minds, and that in truth the Sun is, if not the king, at least the palace of the "intellectual fire"?

I deliberately break off both the sleep and the immense speculation, only exclaiming with the royal psalmist:[184]

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[184] Kepler's hymn is in the spirit of the Psalms of David rather than based on any particular psalm, such as Psalm 48 which begins with the same words. There is, of course, no mention of the planets or celestial harmonies in the psalms.

Great is our Lord, and great is His excellence and there is no count of His wisdom. Praise Him heavens; praise Him, Sun, Moon, and Planets, with whatever sense you use to perceive, whatever tongue to speak of your Creator; praise Him, heavenly harmonies, praise him, judges of the harmonies which have been disclosed; and you also, my soul, praise the Lord your Creator as long as I shall live. For from Him and through Him and in Him are all things, "both sensible and intellectual," both those of which we are entirely ignorant and those which we know, a very small part of them, as there is yet more beyond. To Him be praise, honor, and glory from age to age. Amen.

And you before all, Maestlin, in your happy old age; for you used to enliven these cares with words and with hope.

THE END

This work was completed on the 17/27 May in the year 1618; but Book V was revised (during the time when its printing was in progress) on the 9/19 February 1619.

At Linz, the capital of Upper Austria.

Appendix

Appendix to Book V of *The Harmony*

The Appendix which I promised above in the index to the chapters of Book V is not the one which the reader should expect here:

They start to make a wine jar, but as the wheel turns a water pot emerges.[185]

I shall state the reasons briefly. I was about to navigate a great and overflowing river, inconveniently, when see! the floods of water subsided into a thin stream and a ford appeared, for me to cross on foot, leaving my ship at her mooring, for more necessary purposes. This is what I mean: the three books of Ptolemy's *Harmony*, which I have in Greek in manuscript, along with the commentaries of Porphyry, that very profound philosopher, also in Greek, from the beginning up to Chapter VII of Book II, have also seemed to me worthy of being published complete in Greek and Latin; and for that reason I began ten years ago to translate them into Latin. That translation progressed as far as the middle of the manuscript of Porphyry. I was prevented from making further progress by my change of abode, combined with a great many troubles, and, after I came to Linz, a new start on my astronomical studies. However, when in fact I had proposed a year earlier to issue my five books on *Harmony*, I saw vast weight placed on the comparison of my work with Ptolemy's, particularly of my Book V with the last chapters of Ptolemy's Book III, the last three of which contain nothing but lemmas or headings from Ptolemy. Thus against my inclination I proposed to extract, from a work which was in itself mutilated, only a single part, which was most relevant to my material. To that end I both translated Book III of Ptolemy from Chapter III to the end;[186] and attached to the last three meaningless lemmas their own texts, fitted to them as adroitly as possible according to the basic principles of Ptolemaic astronomy and the intention of their author; and lastly appended commentaries or notes, by which the missing part of the Porphyrian commentary, which breaks off at Chapter VII of Book II, is supplied in this third book, and the Ptolemaic discoveries are compared with my own, and the difference is shown between the

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[185] Horace, *Ars poetica*, 21–22.

[186] It has already been noted that Kepler's translation of part of Book III of Ptolemy's *Harmonica* was first published in the nineteenth century by C. Frisch (KOF, vol. 5, pp. 335–412). The texts of Ptolemy's *Harmonica* and Porphyry's commentary, together with a German translation of the former, have been published by I. Düring in *Göteborgs högskolas Arsskrift*, 36 (1930), no. 1; 38 (1932), No. 2; 40 (1934), No. 1. An English translation of Ptolemy's *Harmonica* is forthcoming in Andrew Barker, *Greek musical writings*, vol. 2.

symbolism of Ptolemy and my own legitimate demonstrations.[187] Hence the weakness and imperfection of his symbolism are shown, and the chief cause of them, that is the falsity of the basic principles of Ptolemaic astronomy, is made clear. Therefore, this appendix alone was going to occupy up to 30 pages of the book; and behold! meanwhile the neighboring Bohemian war broke out, through which not only were the roads blocked but also many of the workmen are enlisting as soldiers. Thus first paper, and then labor which could be used in the war failed us, and finally the time was excessively protracted by the most disagreeable delays. Roused by these obstacles, eventually I took little thought for myself and did away with what I had begun, so that not only did I not bring out the book of that excellent author Ptolemy and his commentator Porphyry, in a mutilated edition of a fragment, but I destroyed it altogether. Moreover, I hesitated over the remaining half of the work in my translation of Porphyry's exposition. Should I write notes on the remaining chapters of Books II and III, and combine them with Porphry's and with those which I already have, written on almost the whole of Book III, so as to complete the work in that way and issue it in Greek and Latin as far as it exists? For that publication another time and another place are required, and a suitable printer.

However, so that nothing whatever of the things I promised above could be missed by the reader, or so that even in this last part nothing should be lacking for the completeness or for the easier understanding of my work on *Harmony*, come, I shall write out the headings of the chapters of Ptolemy's Book III, and compare them with the corresponding points in my own work; and I shall write in summary fashion what is in my notes.

First, what Ptolemy reported in Books I, II, and the beginning of III on the discipline of harmony, I have encompassed in my Book III by another method which is considerably different. However, whereas Ptolemy looks for the basic principles of the harmonies in abstract numbers, along with the ancients, I on the other hand say that there is no force in the numbers as counting numbers, and in their place establish as the basic principles of the harmonies the counted numbers,[188] that is, the things themselves which are subject to the numbers, in other words the plane regular figures and the divisions of the circle which are to be controlled by them; and so I necessarily set Books I and II on the harmonic figures before my Book III.

Now Ptolemy has written at the top of Book III, Chapter III, of his work the title *Under what class of things the nature or force of harmony is to be placed, and the knowledge of it*, showing that there is some prin-

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[187] A summary of the contents of the chapters of Book III relating to the celestial harmonies is given in note 6.

[188] See Book III, note 51.

ciple, causal, formal, mental, or even divine, which links harmonies with things. Chapter IV: *That the force of harmonic combination belongs to all things whose natures stand at a higher degree of perfection; and that that is chiefly apparent in human minds and in the heavenly revolutions.* There you see the division of what remained for Ptolemy to say: that is, first, on the harmonies which exist in minds, second, on the harmonies in the motions of the heaven. And on the first there follow in fact three chapters; on the second the remaining nine. Chapter V compares individual consonances with individual faculties of the rational soul, and does that concisely, by the subdivisions of each of the things compared. Chapter VI compares individual kinds of harmonies with individual kinds or classes of virtues. Chapter VII compares modulations of tones with the modulations of public feeling as times change, and thus with sudden modulations of emotions in minds.

In this part I disagree with Ptolemy on many points, arguing of course that the symbolic associations are for the most part not necessary, or causal, or natural, but rather poetic and rhetorical. For those special faculties and virtues which belong to minds are not allotted either their Idea or the numbers representing their constitution by reason of harmonies, as if from an archetype, but have other admitted and evident causes. Indeed they do not even correspond numerically, but by a far-fetched or overstrained effort an appearance of division into an equal number of parts is imposed on them; many which are surplus, or on the other hand shortages, are disguised; and some which are less appropriate are dragged in by the short hairs. Last, those which do correspond best and most neatly are nevertheless not included on account of some proportion between quantities of the same kind, though harmonies can only subsist in comparisons of quantities.

Yet I showed the comparison between musical tones and public emotions and behavior to be extremely natural, and to be related to their cause, when prompted by the text of Ptolemy I adduced the opinions of various philosophers on the affinity of minds, numbers, and harmonies with each other, to which I also attached the Epilogue to my Book V. Also as far as the specific effects of the tones in moving minds in various ways are concerned, I embraced that matter in particular in Chapter XV of my Book III. Similarly as far as justice of exchange and of distribution, and friendship, and economics and politics are concerned, I made a digression at the end of my Book III on the subject of how far musical proportions are significant for them. Last, I embraced the subject matter of Chapter III and IV, which had already been set out in advance, and the points in Chapters V, VI, and VII in general which could be said to be true, in the first three Chapters and again in Chapter VII of my Book IV. There, after explaining the nature of minds, I showed that since a circle divided geometrically by inscribing the plane figures defines the harmonic proportions truly and appropriately, by comparison of the parts with the whole, therefore, by a particular line of essential reasoning the circle must exist in minds, formally and in the abstract, not only in respect of matter, but also in a way in respect of actual quantity, materially considered. Hence as well as the circle the harmonies also exist in minds, and this is the reason why minds are moved by harmonies. But all this was only in a general way, and no plea followed for the division of the mind into special faculties. Such, therefore, is the position in regard to human minds.

As far as celestial motions are concerned, Ptolemy compares the zodiac with a musical system or scale in Chapter VIII of his Book III, and the actual consonances with the aspects of the planets in Book IX. Here I have shown the particulars in which the theory of aspects in Ptolemy is defective, and how by reestablishing the number of the aspects on one hand and of the consonances on the other there eventually appears a natural and causal connection between the two. However, this is the true subject matter of my Book IV, especially of Chapters IV, V, and VI. In Chapter X Ptolemy produces a sort of melody from the motion of a planet from east to west; in Chapter XI he produces the kinds of music from the true motion of the planets upwards and downwards;[189] in Chapter XII he produces the variety of the musical tones from their motion from side to side of the equator; and in Chapter XIII he produces the different tetrachords of the system from the various configurations of the planets with the Sun. I have shown that in these four chapters Ptolemy runs riot in using poetic or rhetorical comparisons, since what he compares are not real objects in the heaven, with the exception of a small part of Chapter XI, which I wish to be consigned to the last three chapters on account of the affinity of their subject matter. I have also shown that here there is partly a comparison between things which are incompatible, as the nature of what is compared proclaims, and partly things which correspond in all their branches do so merely accidentally, but do not agree by the necessity of any cause. So far, therefore, there is nothing or very little which agrees with my celestial harmonies.

However, Ptolemy's following Chapter, XIV, raises the question, *in terms of what primary numbers can the sounds of the musical system or scale be compared with the primary spheres in the planetary system.* Here I have shown that for Ptolemy and the Pythagoreans the comparison was impossible in their conventional astronomy, though I have attempted to devise something which agrees with the Ptolemaic principles, so that in this way a suggestion of their coherence might be supplied by the bare title. I have also shown that, since the primary spheres depend on their quantities, no less than strings on their lengths, the comparison should have been instituted in accordance with the quantities, even though the proportion of the spheres is extremely repugnant to the proportion of the consonant strings. However, at the be-

[189] That is, their changes in distance.

ginning of my Book V I have also renewed from my *Mysterium Cosmographicum* in corrected form the causal or archetypal link between the spheres of the world and the five regular solids.

Ptolemy's Chapter XV asks *how the proportions of the planets' own motions can be derived through numbers*; but the text is missing. I have, therefore, shown the same as in the preceding chapter. As to the rest, I have presented for consideration the fact that here, 1500 years ago, Ptolemy would have set about handling the subject matter belonging to my Book V, if he had been able to through his own astronomy. Thus I, in the corrected astronomy, which keeps the true and simple motions of the planets, eliminating apparent motions, which depend on optical illusions, have shown that in the heaven according to true and genuine quantitative reasoning, and based on measurement, but not by mere trivial interpretation of symbols, there are all the harmonic proportions, the kinds of harmonies, the musical system or scale, and most of its keys, the varieties of tones, planets which emulate the figured music of voices, and finally the universal counterpoints of the six primary planets, varying both in kinds and in tones. In Chapter XVI which is his last, Ptolemy promises that he intends to investigate *on what basis the family relationships of the wandering stars can be compared with the family relationships of sounds.* That is to say, he has busied himself in deriving the basic principles of astrology, on benign and malignant planets, and those which are friendly or inimical to each other, through the celestial harmonies. And because the text is missing for this chapter also, I have supplied it as far as I could, especially from Macrobius;[190] but at the same time I have shown in the notes what this business lacks for success according to the ancient astronomy. However, in my work I have no section corresponding with Chapter XVI, except for a few lines in the Epilogue to Book V. For astrology deals with the effects of the stars on the Earth; whereas my celestial harmonies are formed by rays, not at the Earth but at the Sun. And this is the end of Ptolemy's work. Along with it I should also bring to a close the appendix promised at the entry to Book V, if the affinity of the subjects did not incite me also to give a satisfaction to those who have been in contention with me, such as Mr. Robert Fludd, the Oxford physician. He filled his book on the Microcosm and the Macrocosm,[191] published a year ago, with reflections on harmony, so that I should not omit to mention

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[190] See note 7.

[191] R. Fludd, *Utriusque cosmi maioris scilicet et minoris, metaphysica, physica atque technica historia* (Openheim, 1617–1618). This work is an attempt to describe the philosophy of the harmonious design of the cosmos and the corresponding harmonies of man. Besides drawing on the Cabala and the Hermetic texts, Fludd also assimilates the ideas of the Rosicrucian philosophy, of which he was one of the chief exponents. The Rosicrucian manifestos, known briefly as the *Fama* and *Confessio*, published in 1614 and 1615 respectively, were followed in 1616 by a strange alchemical romance with the title *Chymische Hochzeit Christiani Rosencreutz*, which has been attributed to Johann Valentin Andreae. For Fludd, harmony meant Pythagorean numerological

him in my book but should briefly show the reader the subjects on which there is agreement between him and myself, and those on the other hand on which we differ.

On the Cosmic Music of Mr. Robert Fludd, the Oxford physician.

That author, then, has promised two volumes, one of which, written on the Macrocosm, has already seen the light, while the second, on the Microcosm, is still awaited. The former volume embraced two treatises, and also brought them out at different times. For I saw the first treatise, on the threefold cosmos, after the autumn Frankfurt Fair of the year 1617, and the second on the Arts, which he calls the apes of cosmic nature, at the Easter Fair of the following year, 1618. In the second treatise, then, he has placed music among the arts, embracing up to a point the subject matter of my Book III. However, in the earlier treatise, which is contained in seven books, he has allotted the third book to cosmic music, taking the same title as I put at the head of my entire work. However, he has taken on the subject matter of my Book IV and Book V. Therefore, we shall start with his artificial music. He tells us of that in seven books, in the first of which he reviews the authorities, the nomenclature, and the force exerted on the minds of men. On the authorities, or the history of the discovery, I have said nothing, or little, inasmuch as my intention is to reveal the causes of things which are natural. The necessary nomenclature I have embraced in my definitions throughout; the superfluous I have omitted. I deal with the force of music in minds in Chapter XV of Book III and throughout Book IV. In the second book the author attacks the actual subject matter, which he speaks of as intervals and times. Again, I have said absolutely nothing about times, or the length or brevity of sounds; for they are arbitrary, and do not need inquiry into causes. He calls certain intervals simple which for me are the smallest melodic dissonances, the major and minor tone, the semitone, and the diesis. He considers the others, which I call consonances, as compounds of these. But I have expressly refuted this opinion of the ancients, that the consonances are composed of smaller intervals which are, so to speak, prior by nature, in Chapter IV of my Book III, showing that the smaller intervals on the contrary arise from the consonant intervals which are larger than themselves. In his third book he expounds the musical system or scale, which is a main part of my Book III from Chapter IV to Chapter IX. The author's remaining four books are on the prac-

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relationships, running through the three worlds of the empyrean, the heavens and the elements, and binding together the macrocosm and the microcosm. But he held alchemy to be the true science, penetrating into the hidden depths of nature. Fludd attacked Kepler in a pamphlet of 1621, claiming that Kepler had treated only the outer movements of nature, while he himself had penetrated the hidden depths. Kepler replied at length in his *Apologia pro opere Harmonices mundi* (1622) (KGW 6, pp. 381–457), rejecting Fludd's Pythagorean numerology and poking fun at his claim to possess deeper perception. When Fludd renewed his attack, Kepler just ignored it. On Fludd, see Yates (1972), pp. 70–90.

tical side, which I do not even touch. For in Book IV he gives advice on the measure of the beat, and on its various modes, and on the value of the notes in them, on which I have a very few points in Chapter XV of Book III and in Chapter III of Book IV. In V he has advice on the composition of figured melody, an art which I do not profess. In VI he also digresses to various musical instruments, to which I had not even given thought. Last, in VII he reveals a new instrument himself. In these last four chapters he differs from me in the way in which a practitioner does from a theorist. For where he writes on instruments, I enquire into the causes of things, or of consonances; and where he gives instruction on composing a tune for several voices, I provide mathematical derivations of many features which occur naturally both in choral and in figured melody. Consequently, there are also many pictures in his work; in mine, mathematical diagrams organized with letters. Notice also that he takes great delight in topics which are hidden in the darkness of riddles, whereas I strive to bring topics which are wrapped in obscurity out into the light of understanding. The former is familiar to chemists, Hermeticists, and Paracelsians; the latter is considered their own by mathematicians.[192]

Furthermore, indeed, in Books II and III, where he is dealing with the same subject matter as myself, this is the difference between us: what he takes over from the ancients, I draw out from the nature of things and establish from the very foundations. He applies what he has got in a confused (on account of the varying opinions of his sources) and uncorrected form: I proceed in the natural order, so that everything is set right according to the laws of nature, and confusion is avoided; so much so that I do not even relate what has been established to the opinions of the ancients, except where no confusion follows. Thus at the point where I expressly refute the ancients' treatment of the causes of consonance, he follows the ancients, without the hazard of hesitation: he does not even give a thought to the truer causes. In a word, in the discipline of harmony, one plays the part of a vocal and instrumental musician, the other of a philosopher and mathematician.

Let us now pass on to another passage of the author's, in which he introduces music into the cosmos. Here the difference between us is of immense size. First, what he endeavors to teach us as harmonies are mere symbolism. Of them I say what I said of Ptolemy's symbolism, that they are poetic or rhetorical rather than philosophical or mathematical. This is the spirit of the whole of this work, as is evident even from the title of Macrocosm and Microcosm. For in the second volume he will undoubtedly endeavor to demonstrate this noble thesis,

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[192] Here Kepler clearly explains the difference between his own strictly mathematical approach and the obscure and confused ideas of the various fanatics of his time such as alchemists, Hermeticists, and Paracelsians.

that the ideas of the whole great cosmos, and of all its parts, are found in man. This same spirit is also that of the first volume, as he divides the whole cosmos into three regions; and there in accordance with the most celebrated axiom of Hermes he makes the *higher things similar* or analogous *to the lower.* However, for this analogy to succeed in all cases, the points of comparison on either side often have to be dragged in by the short hairs. My opinion of analogies is clear from the digression at the foot of my Book III: in other words, although the analogy of proportions in geometry is something formal, in respect of the actual quantitative matter, which is indefinite and undetermined, yet in respect of harmonic proportions it can be considered rather as a material property of the harmonic proportions. For since the harmonic proportions define a certain quantity, analogies on the other hand are apt to extend themselves to infinity, and thus counterfeit the material property of infinity.

However, I also deal with some points similar to what he says on the Microcosm in my work, such as what I say in Book IV. I make the Earth a living creature: but that is for quite different reasons. For I do not contend that there is a pure analogy between the Earth and living things, and neither do I mean that the archetype of a living thing has been taken from the Earth itself; but the proposition which I mean to demonstrate is simply that those works which are seen on the Earth's globe cannot come about from the motions of the elements, or from the properties of matter on their own, but bear witness of the presence of a soul. In that case for my arguments to be understood it was necessary to adduce the various functions of the soul in the body of a living creature. Let us now come closer to the foundations on which Robert Fludd erects his cosmic music. First, he takes possession of the whole cosmos and all its three parts, empyrean, celestial, and elementary: I, of the celestial alone, and not the whole of it, but only of the motions of the planets, so to speak, against the zodiac. He trusts the ancients, who believed that the force of the harmonies comes from abstract numbers, and considers it sufficient if he demonstrates that there is consonance between any of the parts, in whatever way he expresses them in numbers, not caring what sort of units are combined together in that number: I teach that harmonies should never be sought when the things between which the harmonies are cannot be measured by the same quantitative measure, in such a way that with respect to quantity the proportion between them is the same as there is with respect to length between two strings at the same tuning. Consequently, he divides the whole cosmos into three equal parts by means of a radius, taking it as sufficiently well known that they are far from equal, but for the sole reason that the first unit is the elementary world, the second the aethereal world, and the third the empyrean. And in fact the units cannot be depicted otherwise than by equality of lines. But I, unless astronomy bears witness that the units share the same quantitative measure, do not in any way employ them as units for numbering the harmonic proportions. He, however, standing on his principles, and erecting a pyramid on the great circle of the Earth as base, sets its vertex at the very apex of the empyrean heaven; and dividing its height into three equal sections (just as if he had in absolute truth had equal units), he counts how many parts belong to the empyrean, how many to the celestial, how many to the elementary. For the top of the elementary region on this showing is twice as far away from the top of the celestial region as from the top of the empyrean; and in the division of the pyramid, with respect indeed to the axis, three equal sections belong to the three regions, but with respect to the triangle on the axis, one unit belongs to the empyrean, three units to the celestial, and five to the elementary. Finally, with respect to volume or fatness of shape, one unit belongs to the empyrean, seven units to the celestial, and nineteen to the elementary. Now what shall I say about the other, contrary pyramid of light, of which he makes the worshipful Trinity itself the base, at the topmost apex of the empyrean heaven, and places the vertex on the very Earth? Since he mingles these two pyramids with each other, and elicits musical proportions from the mixture, he is attempting something entirely different from the intention of my work. For he compares light (which bestows form and spirit) and matter, two things which are completely different from each other, and to which quantities do not in any way belong in the same respect; but I admit, as terms in forming harmonic proportion in the universe, only those things which admit quantities in the same respect, for instance the motion of Mars and the motion of Jupiter, both diurnal. The difference between us consists equally of this fact also, that he ascribes to the elementary region four degrees of obscurity and darkness, *because*, he says, *everything has four quarters*, certainly no less than three thirds or five fifths; and in fact all four belong to the Earth, three to water (and therefore, it is in fact transparent), two to air, and one to fire. Again elsewhere he subdivides every region, belonging either to an element or to a heaven, into three spaces, top, middle, and bottom, which the agreement of the senses does not follow in every case. You see that his units are again arbitrary. However, he proceeds on that account to establish a diatessaron between Earth and water, and to relate its three intervals, a tone, a tone and a semitone, to the three spaces, top, middle, and bottom, since the former have definite quantities arising from their causes, the latter not even boundaries from nature, but measures which are plainly indeterminate drawn from these very general principles; and so on. But I have set out units which are natural, that is to say the two extreme motions of each planet (whether diurnal or hourly makes no difference), expressed by their nature in their definite quantities, in which to seek harmonies. He seeks harmonic proportions in degrees of darkness and light, without respect to any motion: I seek harmonies only in motions. He plucks out a few trivial consonances, and elicits them from the mixture of his pyramids, from which he conjures up the cosmos privately depicted in his mind, or deems them to be represented by it. I have demonstrated that the whole body of harmonic combinations, with all its parts, is found in the planets' own extreme motions, according to measures which are certain and derived from astronomy. Thus for him his conception of the cosmos, for me the cosmos itself, or the real motions of the planets in it, are the basis of the cosmic harmony.

From this short discussion I think it is established that, although knowledge of the harmonic proportions is absolutely necessary for understanding the crowded secrets of the deepest philosophy, of which Robert tells, yet even if he has thoroughly learnt the whole of my work, he will still be considerably further from those most intricate secrets; and the proportions have departed from the totally accurate certainty of mathematical derivations. And now let this also be the end of the Appendix.

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Abbreviations

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